Scheduling Doctors to Clinical and Surgical Time Slots: A Column Generation Approach

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Outline

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Model Formulation

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The Problem

Develop an application to take:

- Requests for surgical spaces (a cardiology surgery, at Location 1, in the AM, two days a week, two weeks per planning horizon)
- Requests for clinical spaces (an ENT clinic, at Location 2, all day, Monday and Wednesday, every week)
- Doctor requests (I play golf every Tuesday afternoon)

Find a schedule:

- Match available surgery and clinic rooms at the various locations with the space requests
- Match doctor requests as closely as possible to the space request assignments

The Problem

- Administrative assistants for each specialty need to sit at their desktop machines and enter the space and doctor request information for the given specialty.
- The requests across all specialties must be gathered and stored somewhere.
- An optimization model must be built and solved that finds a feasible allocation of space requests to available rooms and doctors to meet the requests
- The results need to be returned to the desktops of each administrative assistant in order to schedule the doctors.
- The system must be run and used every month by people that know nothing about optimization.

System Architecture

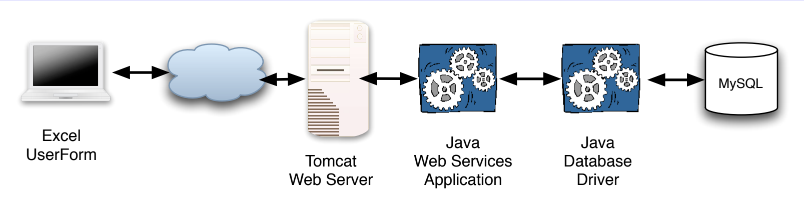


Figure: Entering the Data.

System Architecture

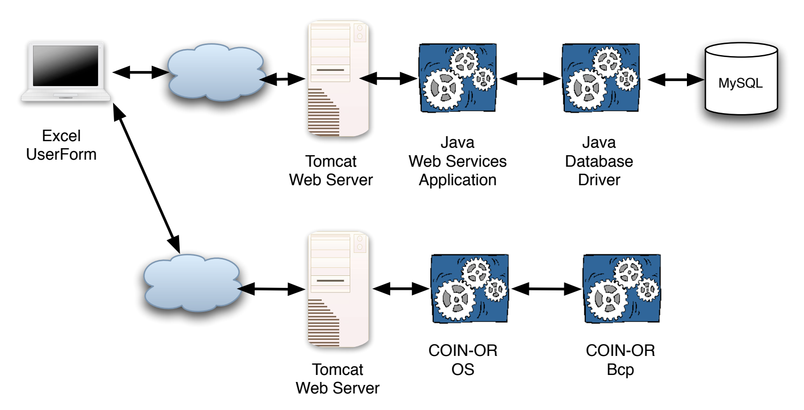


Figure: Calling the Scheduling Application

System Architecture

- Excel (Client User Form)
- Visual Basic (Create SQL requests from User Form)
- MySQL (Relational Data Base)
- **COIN-OR Bcp Solver**
- **COIN-OR OS**
- HTTP / Tomcat
- JDBC (The database drivers)
- Java (Web Services)

Key Concepts

- **Space:** a room or set of rooms (*pod*) that can service either a clinic or surgery visit – each space has a capacity
- **Space Request:** a specification of
  - location
  - type – surgery or clinic
  - specialty
  - weeks
  - days
  - shifts (AM or PM)
  - number of spaces

Key Concepts

Assignment: A complete specification weeks, days, shifts, and spaces that satisfy a request for a clinic or surgery space at a location

Request	Assignment
Liberty	Liberty
Surgery	Liberty
ENT	ENT
one week	second week
two days	Wed, Fri
either shift	AM (Wed), PM(Fri)
2 spaces	Room 212

Request Slot: a specification of location, week, day, shift, space(room) ID, request type (surgery or clinic). **As assignment is a specification request slots.**

Example of Request and Request Slots

A request:

```
ENT,Surgical Space,Liberty, A.M., 3 days, 2 weeks
```

This request generates an assignment that specifies the following request slots:

```
ENT,Liberty,Surgical,Storz,Week 1,M,AM
ENT,Liberty,Surgical,Storz, Week 1,W,AM
ENT,Liberty,Surgical,Storz, Week 1,F,AM
ENT,Liberty,Surgical,Stryker, Week 2,M,AM
ENT,Liberty,Surgical,Stryker, Week 2,W,AM
ENT,Liberty,Surgical,Stryker, Week 2,F,AM
```

Note: We are not worried about doctors right now.

Model Formulation

Indexes:

- i – index of requests
- j – index for assignments
- k – index of request slots

Parameters:

n – number of requests

m – number of slots

d – number of dummy slots (every location has a dummy surgery room and a dummy clinic room)

S_i – an index set of assignments for request i

c_k – capacity of room in slot k

a_{ijk} – number of spaces allocated to *request slot* k in assignment j of request i

Model Formulation – Space Requests Only

Variables:

- x_{ij} –1 if assignment j is selected for request i
- z_k – number of spaces allocated to dummy slot k, each location has a "dummy" space for each time slot (day, week, shift).

$$
\begin{aligned}
\min \quad & \sum_k z_k && && (1)\\
\text{s.t.} \quad & \sum_{i=1}^{n} \sum_{j \in S_i} a_{ijk} x_{ij} \leq c_k, && k = 1, \ldots, m && (2)\\
& \sum_{i=1}^{n} \sum_{j \in S_i} a_{ijk} x_{ij} \leq z_k, && k = m+1, \ldots, m+d && (3)\\
& \sum_{j \in S_i} x_{ij} = 1, && i = 1, \ldots, n && (4)\\
& x_{ij} \in \{0, 1\}, && j \in S_i, \quad i = 1, \ldots, n && (5)
\end{aligned}
$$

Model Formulation – Now Add Docs

Key Doc Concept 1 – a **doc slot:** a specification of location, week, day, shift, request type (surgery or clinic)

Example Doc Slot: Liberty, clinic, second week, Tuesday, AM, cardiology

A *doc slot* is like a *request slot* except that it **does not** have a room specification, but **does have** a specialty.

Slot Summary

Request Slot:

- week
- day
- shift
- location
- clinic or surgery
- **room**

Doc Slot:

- week
- day
- shift
- location
- clinic or surgery
- **specialty**

Slot Summary

Slots Continued:
Time Slot:

- week
- day
- shift

Model Formulation – Now Add Docs

Key Doc Concept: a **doc feasibility vector:** a list of indexes of *doc slots* a doctor *is available* to fill. Note: The doc slot might correspond to clinic, it might correspond to surgery.

Each doc has a feasibility vector.

The feasible vector is found for each doc by looping over the doc requests for that doc.

Model Formulation – Building Doc Feasibility Vector

- Education
- Research
- Academic
- Unavailable (specific location or all)
- Specific Clinic
- Specific Surgery

So doc requests have two flavors: when the doc cannot do a clinic or surgery (education, research, academic, unavailable) and when the doc would like to do either a clinic or surger.

Model Formulation – – Building Doc Feasibility Vector

For `Specific Clinic Request Any Location` – eliminate all *doc slots* for surgery in that time slot at every location. Do not eliminate any *doc slots* for clinics in any location and makes these variables more desirable in the objective function.

For `Specific Clinic Request Specific Location` – eliminate all *doc slots* for surgery in that time slot and every location. Eliminate all *doc slots* for clinics in locations other than the specified location

Do a similar thing for Specific Surgery Requests.

Model Formulation – With Docs

Additional Indexes:

h – index of docs, $h = 1, \ldots, q$

l – index of doc slots, $l = 1, \ldots, p$

Additional Parameters:

q – number of docs

p – number of doc slots

b_{hl} is 1 if *doc slot* l is an index in the doc h feasibility vector (actually we do not define a variable y_{hl} when $b_{hl} = 0$)

θ_l is an index set of (*specialty*, *request slot*) pairs that map onto *doc slot* l, $l = 1, \ldots, p$

Model Formulation – With Docs

Example: (Specialty, Request slot) pairs

```
(ENT, Liberty, surgery, second week, Tuesday, AM, Pod 1)
(ENT, Liberty, surgery, second week, Tuesday, AM, Pod 2)
```

map onto *doc slot*

```
(ENT, Liberty, surgery, second week, Tuesday, AM)
```

implemented using C++ STL Container Map.

Model Formulation – With Docs

So for each assignment j in request i (the request determines the specialty) we sum up over all the $(specialty, k)$ pairs that map to doc slot l

$$\alpha_{ijl} = \sum_{k \in \theta_l} a_{ijk}$$

Model Formulation – With Docs

Additional Variables:

y_{hl} – 1 if doc h assigned *doc slot* l, 0 otherwise

Additional Constraints:

$$\sum_{i=1}^{n} \sum_{j \in S_i} \alpha_{ijl} x_{ij} \leq \sum_{h=1}^{q} b_{hl} y_{hl}, \quad l = 1, \ldots, p \tag{6}$$

Model Formulation – With Docs

Finally, one more a constraint. In each *time slot* a doc can only have assignment. Let t index time slots.

Additional Constraints:

$$\sum_{l=1}^{p} d_{tl} y_{hl} \leq 1, \quad t = 1, \ldots, T, \quad h = 1, \ldots, q \tag{7}$$

where $d_{tl} = 1$ if time slot t coincides with *doc slot* l.

For example, the *doc slot*

`ENT, Liberty, clinic,  second week, Tuesday, AM`

corresponds with *time slot*

`second week, Tuesday, AM`

Model Formulation – With Docs

Critical ideas:

1. **Do NOT** solve (1)-(7) for all columns.
2. Solve a restriction of (1)-(7).
3. Generate all y variables up front
4. Generate x variables with column generation

Column Generation

The are less than 100 thousand rows. Not a big deal.

What about variables? Let's just think about a **single request** and *how many assignments might be possible to satisfy the single request.*

The request is for either shift, every day, every week and there are two rooms available at the requested location that can handle this request. *How many columns do we need in the model* to handle this?

There are 25 days in the planning horizon and each day there are 4 choices (2 shifts and 2 room). The number of assignments is

$$4^{25} = 2^{50} \approx 10^{15}$$

CCH – Number of Assignments

Let:

- m – number of weeks requested
- n – number of days requested
- s – number of spaces that satisfy the request

$$\left(\frac{5!}{m!(5-m)!}\right)\left(\frac{5!}{n!(5-n)!}\right)^m (2^n)^m (s^n)^m$$

A request for either shift, every day, every week with two rooms available at the requested location that can handle the request corresponds to $m = 5$, $n = 5$, and $s = 2$.

Column Generation

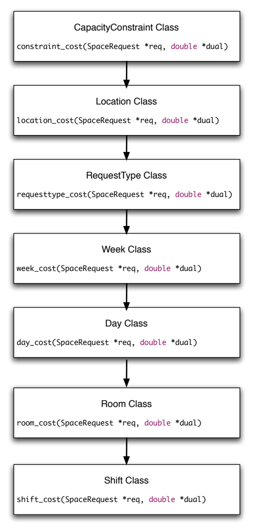
The column generation makes heavy use of object oriented programming. We have a class for each dimension:

- Location class
- Request Type class (surgery or clinic)
- Week class
- Day class
- Room class
- Shift class (AM or PM or Both)

We organize these classes in a hierarchy.

Column Generation

The Class structure used for pricing:



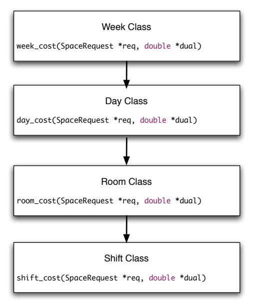
Column Generation

Amount of work: Recall pricing a request for a location that has two rooms that can handle the request. The request is for either shift, every day, every week and would require pricing out 2^{50}.

$$4 + 4 + 4 + \ldots + 4 + 4 = 4 \times 25 = 100 \text{ operations}$$

Order Matters: Working with these objects is like working with Lego blocks. I can put them together different ways. For example, in the current setup, I can schedule an ortho surgery in a different room at Liberty every day of the week. What if I want surgery in the same room every day?

Column Generation



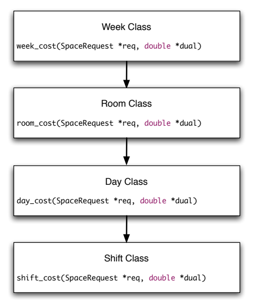
The following assignments:

```
Assignment 776,Liberty,Surgery,Stryker,1,M,AM,1
Assignment 776,Liberty,Surgery,Storz,1,T,AM,1
Assignment 777,Liberty,Surgery,Storz,1,M,AM,1
Assignment 777,Liberty,Surgery,Stryker,1,T,AM,1
```

satisfy the requests:

```
Request 776,Ortho,Surgery,Liberty,AM,M,T,Week 1
Request 777,ENT,Surgery,Liberty,AM,M,T,Week 1
```

Column Generation



The following assignments:

```
Assignment 776,Liberty,Surgery,Stryker,1,M,AM,1
Assignment 776,Liberty,Surgery,Storz,1,T,AM,1
Assignment 777,Liberty,Surgery,Storz,1,M,AM,1
Assignment 777,Liberty,Surgery,Stryker,1,T,AM,1
```

are not allowed for the requests:

```
Request 776,Ortho,Surgery,Liberty,AM,M,T,Week 1
Request 777,ENT,Surgery,Liberty,AM,M,T,Week 1
```

Column Generation Results

Problem	Potential Columns	Root Node Columns	Total Generated	Nodes
1	9,154	62	172	117
2	9,893	29	145	123
3	20,978,496,312	502	3,092	1,341

A Key Takeaway: With Bcp you have **CONTROL** over the branch-and-bound tree in terms of adding cuts or columns.