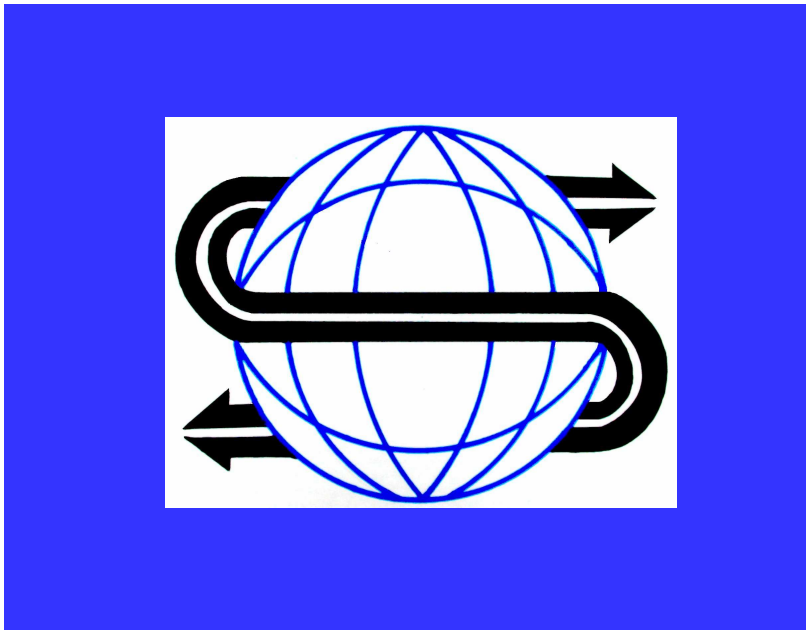

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An XML-based schema for stochastic programs

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Outline

- Motivation
- Stochastic programs: Dynamic and stochastic structure
- Taxonomy of problems
- A three-stage investment problem
- OSiL format
- Stochastic extensions
- Conclusions and future work

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Motivation

- General format for as wide a variety of problem instances as possible
- Essential for benchmarking, archiving, algorithm development
- Useful for distributed computing
- Nonlinear capabilities

Stochastic programs

$$
\begin{array}{lll}
\min & f_0(x_0) + f_1(x_0, x_1) + \mathrm{K} + f_T(x_0, x_1, \mathrm{K}\ , x_T) & \\
\text{s.t.} & G_0(x_0) & \sim b_0 \\
 & R_1(x_0) & \Delta\ r_1 \\
 & G_1(x_0, x_1) & \Delta\ b_1 \\
 & \mathrm{M} & \mathrm{M} \\
 & G_T(x_0, x_1, \mathrm{K}\ , x_T) & \Delta\ b_T
\end{array}
$$

$$l_0 \le x_0 \le u_0$$

$$l_t \le x_t \le u_t, t = 1, \mathrm{K}\ , T$$

Any data item with nonzero subscript may be random (including dimensions where mathematically sensible)
~ stands for arbitrary relation (≤, =, ≥)

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Constraints involving random elements

$$G_t(x_0, x_1, \mathrm{K}, x_t) \,\Delta\, b_t$$

Δ means ~ with probability 1

or with probability at least β

or with expected violation at most v

or …

Problem classes

- Recourse problems
  - All constraints hold with probability 1
- Chance-constrained problems
  - Typically single stage
- Hybrid problems
  - Recourse problems including features such as chance constraints or integrated chance constraints

Dynamic and stochastic structure

- Dynamic structure
  - Periods/stages
- Stochastic structure
  - Independent random variables
  - Period-to-period independence
  - Scenario tree
  - Factor models
  - ARMA processes
  - Trap states and stochastic problem dimensions

A sample problem

$$\min \quad c_0 x_0 + h_0 y_0 + \sum_{s=1}^{S} p_{1s}(c_1 x_{1s} + h_1 y_{1s}) + \sum_{s=1}^{S}\sum_{r=1}^{R_s} p_{1s} p_{2sr}(c_2 x_{2sr} + h_2 y_{2sr})$$

s.t.

$$\begin{aligned}
A_0 x_0 &\leq b_0 \\
x_0 - y_0 &= d_0 \\
v y_0 &\leq K \\
A_1 x_{1s} &\leq b_{1s}, \quad s = 1, \mathrm{K}\ , S \\
y_0 + x_{1s} - y_{1s} &= d_{1s}, \quad s = 1, \mathrm{K}\ , S \\
v y_{1s} &\leq K \\
A_2 x_{2sr} &\leq b_{2sr}, \quad s = 1, \mathrm{K}\ , S, r = 1, \mathrm{K}\ , R_s \\
y_{1s} + x_{2sr} &\geq d_{2sr}, \quad s = 1, \mathrm{K}\ , S, r = 1, \mathrm{K}\ , R_s \\
x_0, y_0, x_{1s}, y_{1s}, x_{2st}, y_{2st} &\geq 0
\end{aligned}$$

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Example (Birge)

$$\max \sum_{s=1}^{S} p_s (w_s - \beta u_s)$$

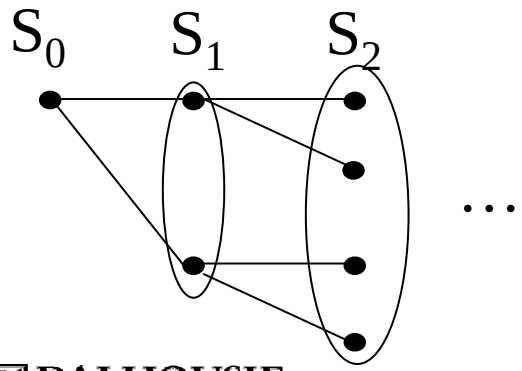
$$\text{s.t.} \quad \sum_{i=1}^{I} x_{0i} = B$$

$$\sum_{i=1}^{I} \alpha_{0is} x_{0i} - \sum_{i=1}^{I} x_{1is} = 0, s \in S_1$$

$$\sum_{i=1}^{I} \alpha_{t-1,i,s} x_{t-1,i,a(s)} - \sum_{i=1}^{I} x_{tis} = 0, s \in S_t, t = 2, \mathrm{K}, T-1$$

$$\sum_{i=1}^{I} \alpha_{T-1,i,s} x_{T-1,i,a(s)} + u_s - w_s = R, s \in S_T$$

$$x_{tis}, u_s, w_s \geq 0$$



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SMPS format

- Three files based on MPS format
  - Core file for deterministic problem components
  - Time file for dynamic structure
  - Stoch file for stochastic structure
- Disadvantages
  - Old technology
  - Limited precision (12 digits, including sign)
  - Limited name space (8 characters)
  - Direction of optimization (min/max) ambiguous
  - Linear constraints, quadratic objective only

Example

$I = 2$, $T = 3$, $B = 55$, $R = 80$, $\alpha_{t1} = \{1.25, 1.06\}$, $\alpha_{t2} = \{1.14, 1.12\}$

Core file

```
ROWS
    Budget0
    Object
    Budget1
    Budget2
    Budget3
COLS
    X01       Budget0    1.0
    X01       Budget1    1.25
    ...
RHS
    rhs1      Budget0    55.
    rhs1      Budget3    80.
ENDATA
```

Stoch file

```
BLOCKS          DISCRETE
 BL Block1                    0.5
    X01        Budget1        1.25
    X02        Budget1        1.14
 BL Block1                    0.5
    X01        Budget1        1.06
    X02        Budget1        1.12
 BL Block2                    0.5
    X11        Budget2        1.25
    X12        Budget2        1.14
    ...
ENDATA
```

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Algebraic modelling languages

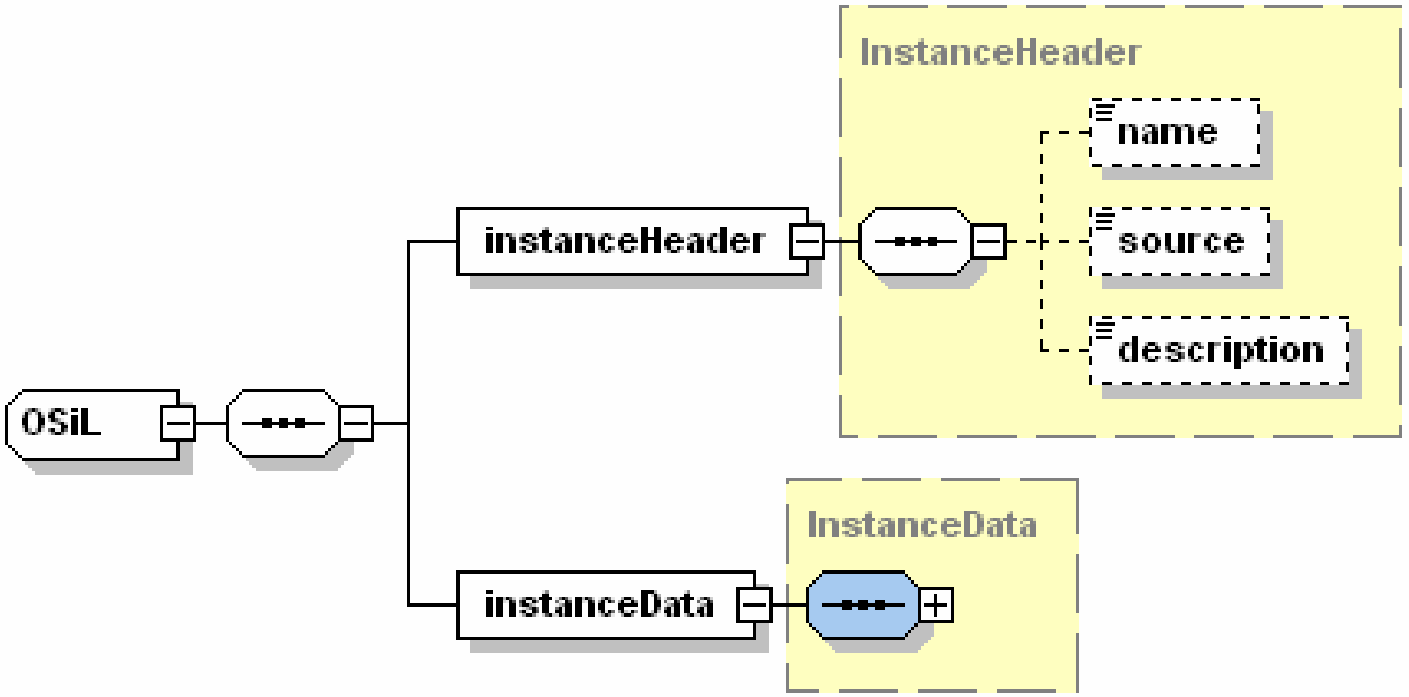
- Characteristics
  - Similar to algebraic notation
  - Powerful indexing capability
  - Data verification possible
- Disadvantages
  - Discrete distributions only
  - Limited consistency checks for stochastic structure

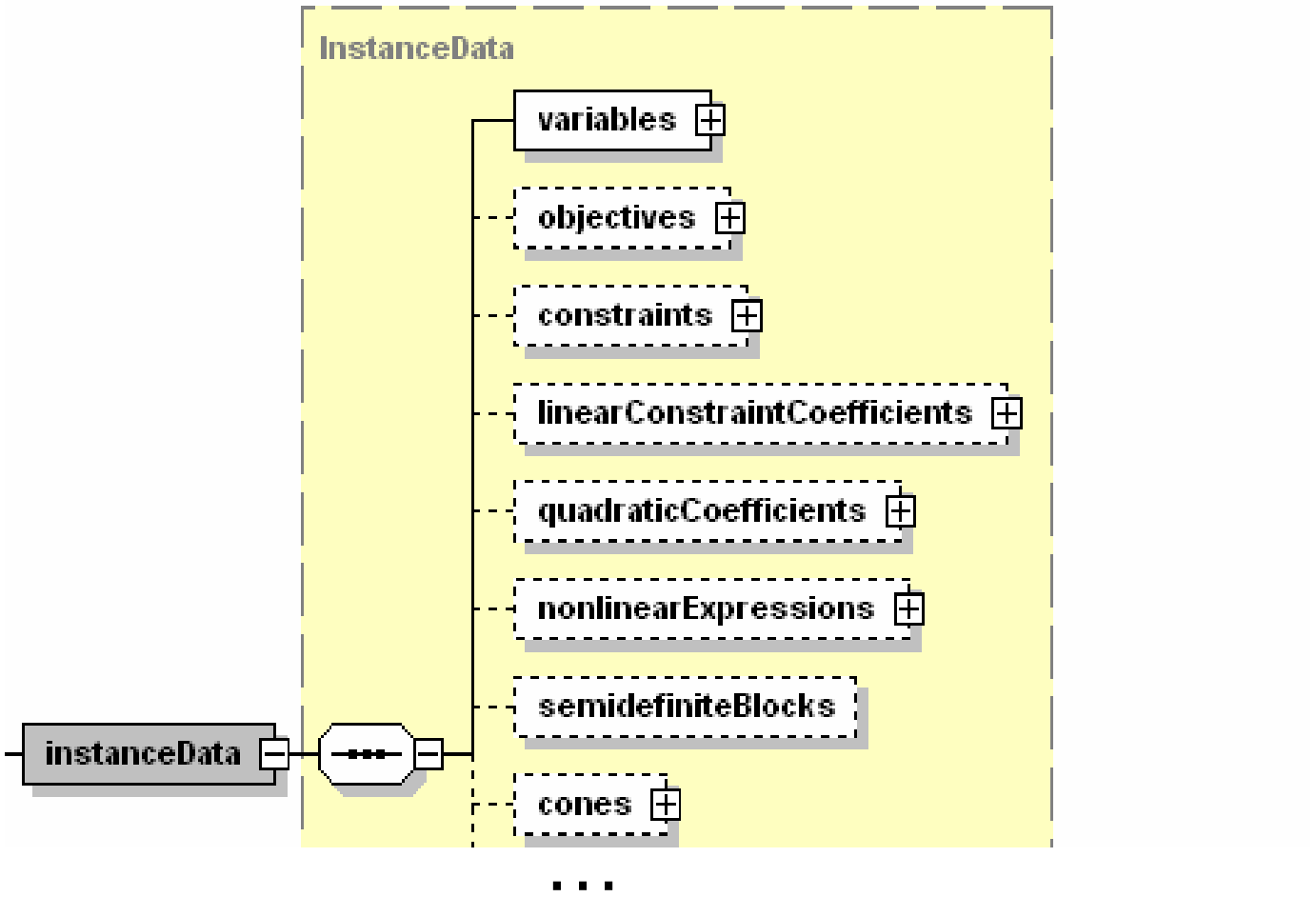
AMPL model

```
param T;
param penalty;
param budget;
param target;
set instruments;
set scenarios;
param prob{scenarios};
set slice{t in 0..T} within scenarios;
param ancestor {t in 1..T, s in slice[t]};
var over {slice[T]};
var under{slice[T]};
param return {t in 1..T,  i in instruments,s in slice[t]};
var   invest {t in 0..T-1,i in instruments,s in slice[t]};

maximize net_profit:
   sum{s in scenarios} prob[s]*(over[s] - penalty*under[s]);

subject to wealth{t in 0..T, s in slice[t]}:
(if t < T then sum{i in instruments} invest[t,i,s]) =
(if t = 0 then budget
          else sum {i in instruments}
                     return[t,i,s]*invest[t-1,i,ancestor[t,s]]
          + if t = T then under[s] - over[s] + target);
```


OSiL – Optimization Services instance Language

- Written in XML
  - Easy to accommodate new features
  - Existing parsers to check syntax
- Easy to generate automatically
- Trade-off between verbosity and human readability
- Stochastic extensions for dynamic and stochastic structure





OSiL – Header information

```
<?xmlversion="1.0"encoding="UTF8"?>
  <OSiL xmlns="os.optimizationservices.org"
    xmlns:xsi=http://www.w3.org/2001/XMLSchemainstance
    xsi:schemaLocation="OSiL.xsd">
   <programDescription>
    <source>FinancialPlan_JohnBirge</source>
    <maxOrMin>max</maxOrMin>
    <objConstant>0.</objConstant>
    <numberObjectives>1</numberObjectives>
    <numberConstraints>4</numberConstraints>
    <numberVariables>8</numberVariables>
   </programDescription>
   <programData>
     ...
   </programData>
  </OSiL>
```


OSiL – Program data – Constraints and variables

```
<constraints>
  <con name="budget0" lb="55" ub="55"/>
  <con name="budget1" lb="0" ub="0"/>
  <con name="budget2" lb="0" ub="0"/>
  <con name="budget3" lb="80" ub="80"/>
</constraints>
<variables>
  <var name="invest01" type="C" lb="0.0"/>
  <var name="invest02"/>
  <var name="invest11"/>
  <var name="invest12"/>
  <var name="invest21"/>
  <var name="invest22"/>
  <var objCoef="1" name="w"/>
  <var objCoef="4" name="u"/>
</variables>
```

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OSiL – Core matrix (sparse matrix form)

```
<coefMatrix>
 <listMatrix>
  <start>
   <el>0</el>
   <el>2</el>
   <el>4</el>
   <el>6</el>
   <el>8</el>
   <el>10</el>
   <el>12</el>
   <el>13</el>
  </start>
```

```
<rowIdx>
   <el>0</el>
   <el>1</el>
   <el>0</el>
   <el>1</el>
   <el>1</el>
   <el>2</el>
   <el>1</el>
   <el>2</el>
   <el>2</el>
   <el>3</el>
   <el>2</el>
   <el>3</el>
   <el>3</el>
   <el>3</el>
 </rowIdx>
```

```
<value>
   <el>1</el>
   <el>1.25</el>
   <el>1</el>
   <el>1.14</el>
   <el>1</el>
   <el>1.25</el>
   <el>1</el>
   <el>1.14</el>
   <el>1</el>
   <el>1.25</el>
   <el>1</el>
   <el>1.14</el>
   <el>1</el>
   <el>1</el>
 </value>
```

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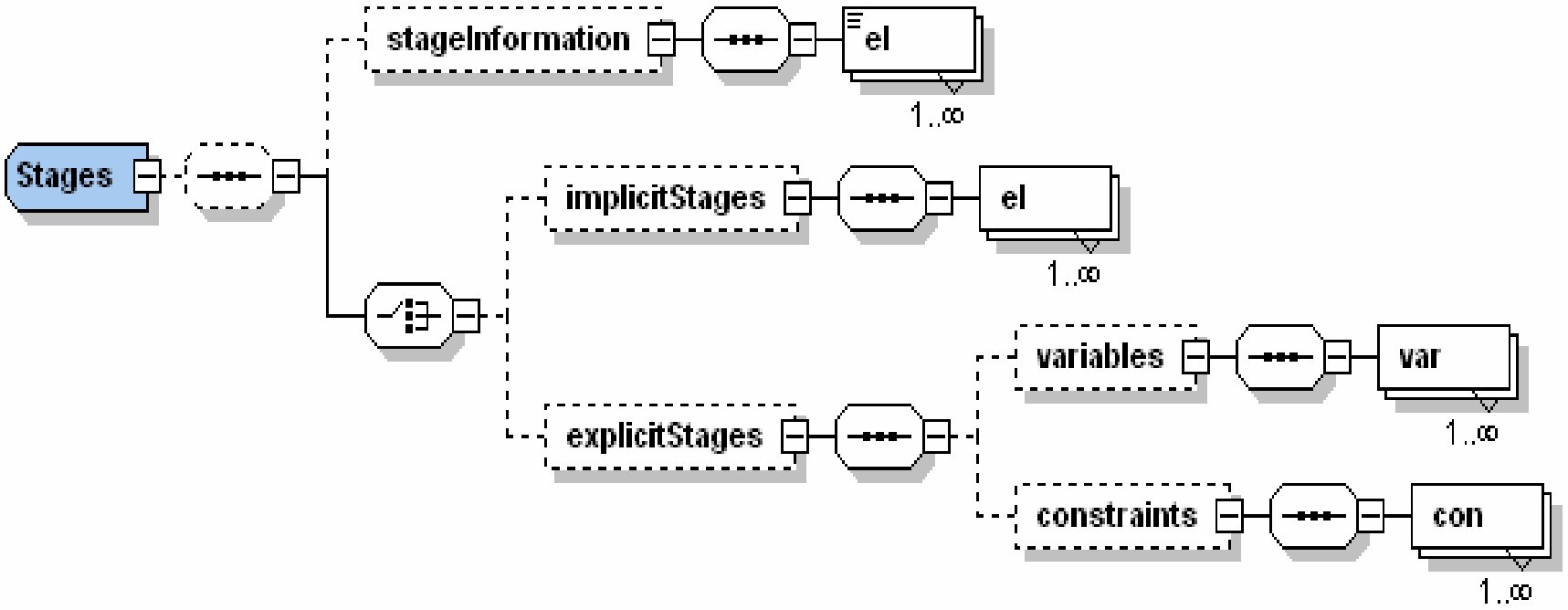
Dynamic structure



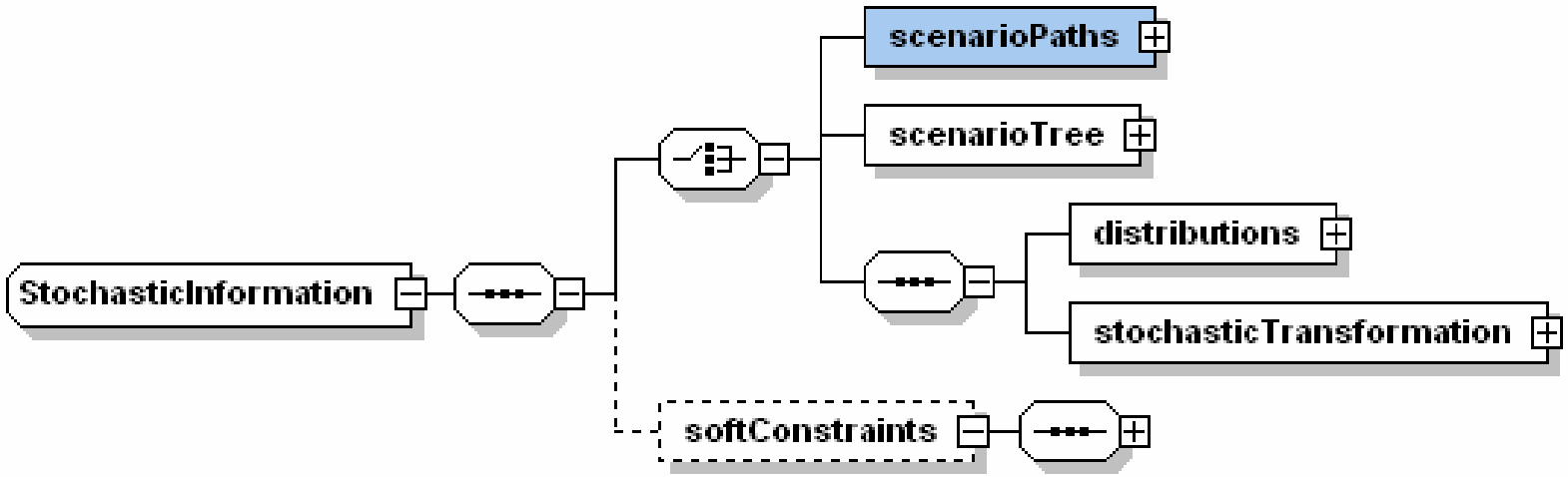
What is a stage?

- Stages form a subset of the time structure
- Stages comprise decisions and events
- Events must either precede all decisions or follow all decisions
- Should a stage be *decision – event* or *event – decision*?

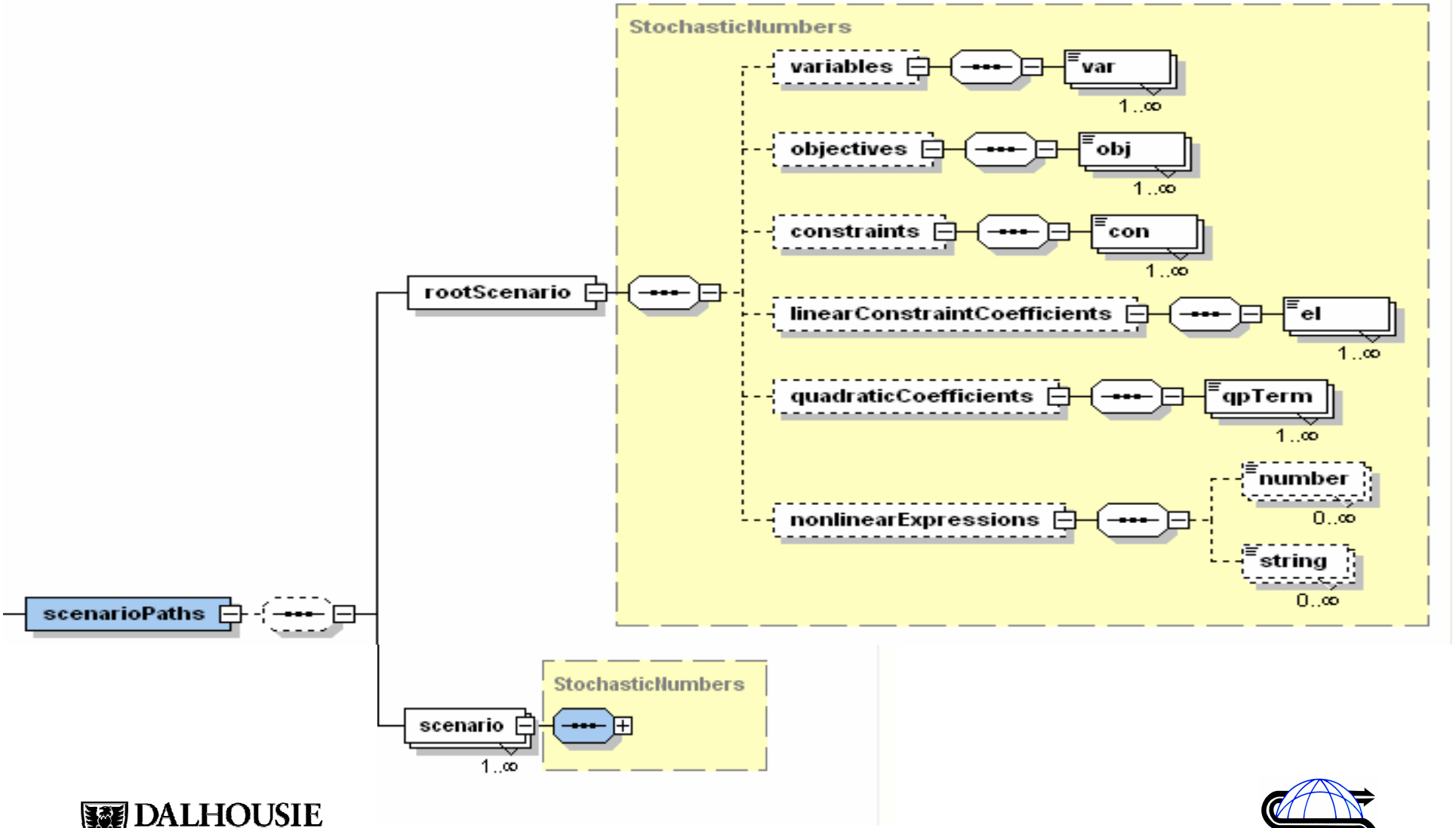
OSiL – dynamic information

```
<stages number="4" order="decisionEvent" >
  <implicitOrder>
    <el startRowIdx="0" startColIdx="0"/>
    <el startRowIdx="1" startColIdx="2"/>
    <el startRowIdx="2" startColIdx="4"/>
    <el startRowIdx="3" startColIdx="6"/>
  </implicitOrder>
</stages>
```


Explicit and implicit event trees



Scenario trees



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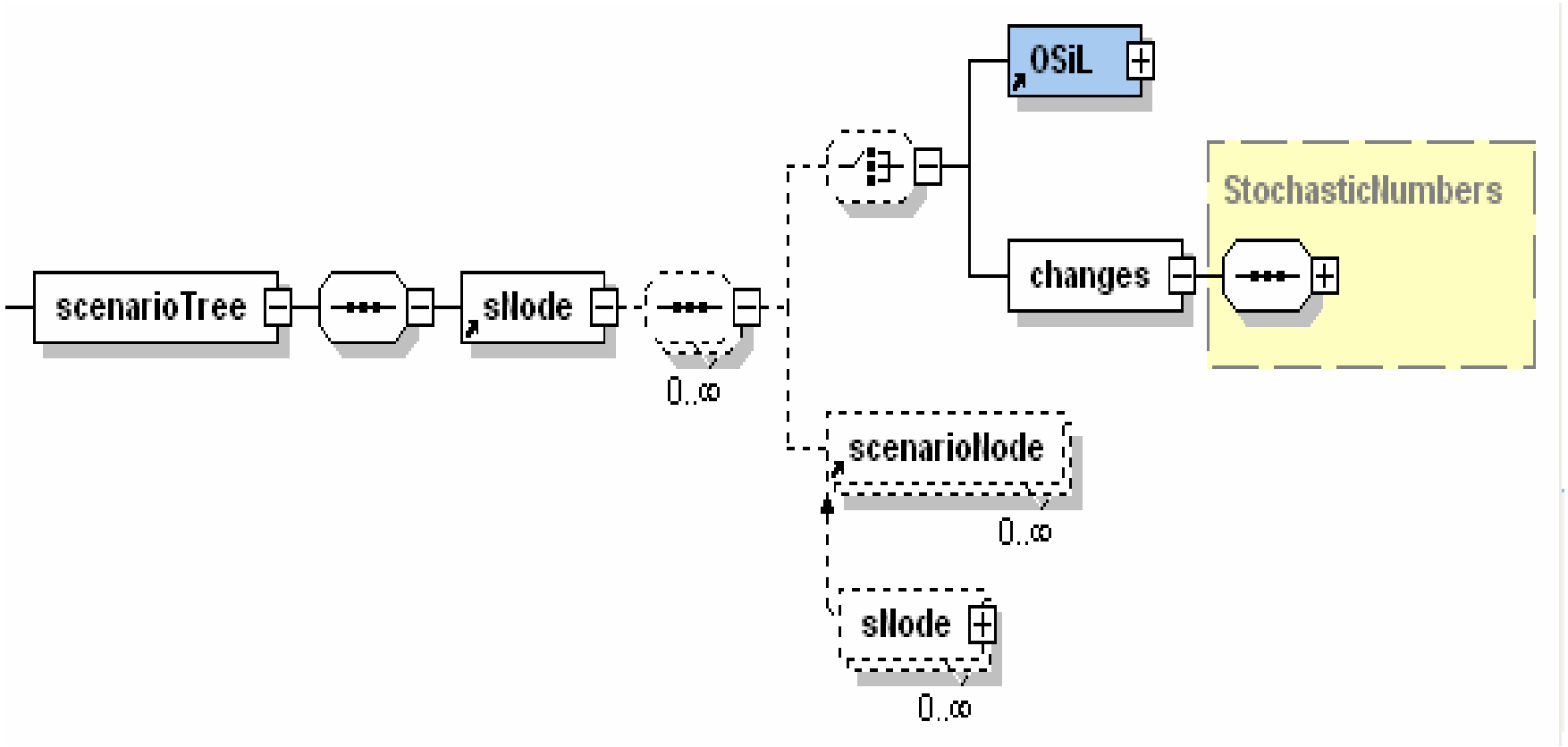
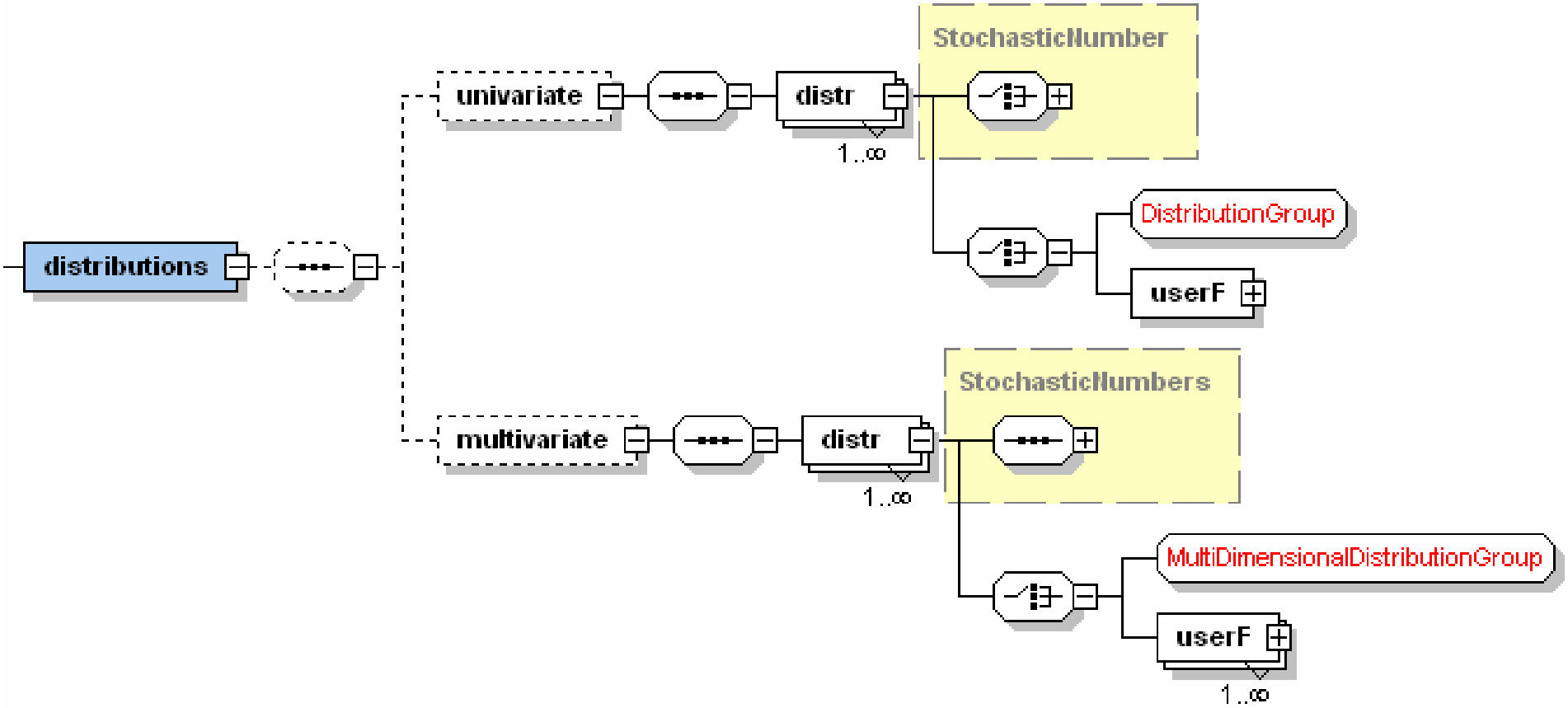
OSiL – Stochastic information

```
<stochastic>
 <explicitScenario>
  <scenarioTree>
   <sNode prob="1" base="coreProgram">
    <sNode prob="0.5" base="coreProgram">
     <sNode prob="0.5" base="coreProgram">
      <sNode prob="0.5" base="coreProgram"/>
      <sNode prob="0.5" base="firstSibling">
       <changes>
        <el rowIdx="3" colIdx="4">1.06</el>
        <el rowIdx="3" colIdx="5">1.12</el>
       </changes>
      </sNode>
     </sNode>
    ...
```


Node-by-node representation for stochastic problem dimensions



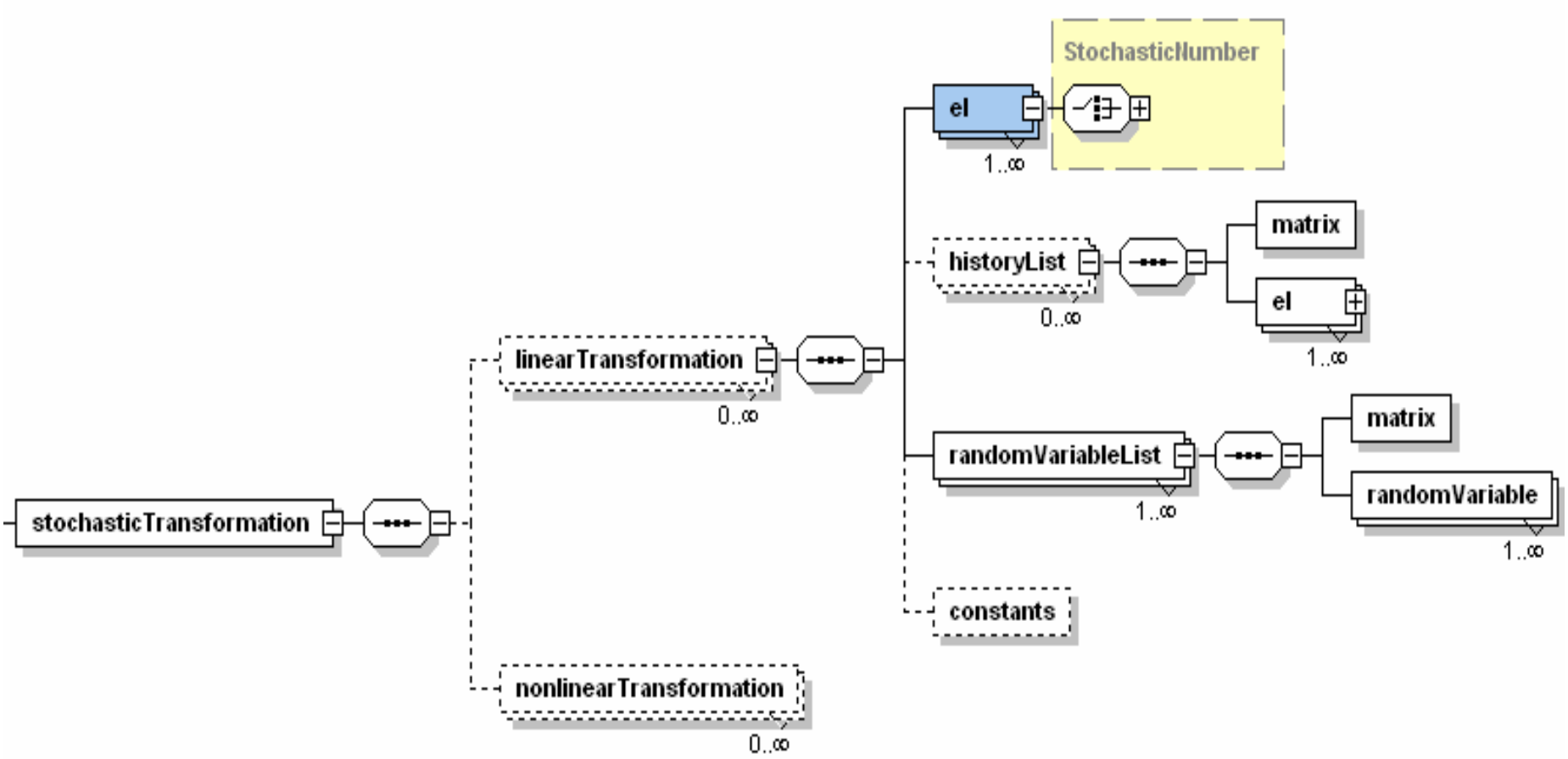
Distributions



OSiL – discrete random vector

```
<distributions>
  <multivariate>
    <dist name="dist1">
      <multivariateDiscrete>
        <scenario>
          <prob>0.5</prob>
          <el>1.25</el>
          <el>1.14</el>
        </scenario>
        <scenario>
          <prob>0.5</prob>
          <el>1.06</el>
          <el>1.12</el>
        </scenario>
      </multivariateDiscrete>
    </dist>
  </multivariate>
</distributions>
```

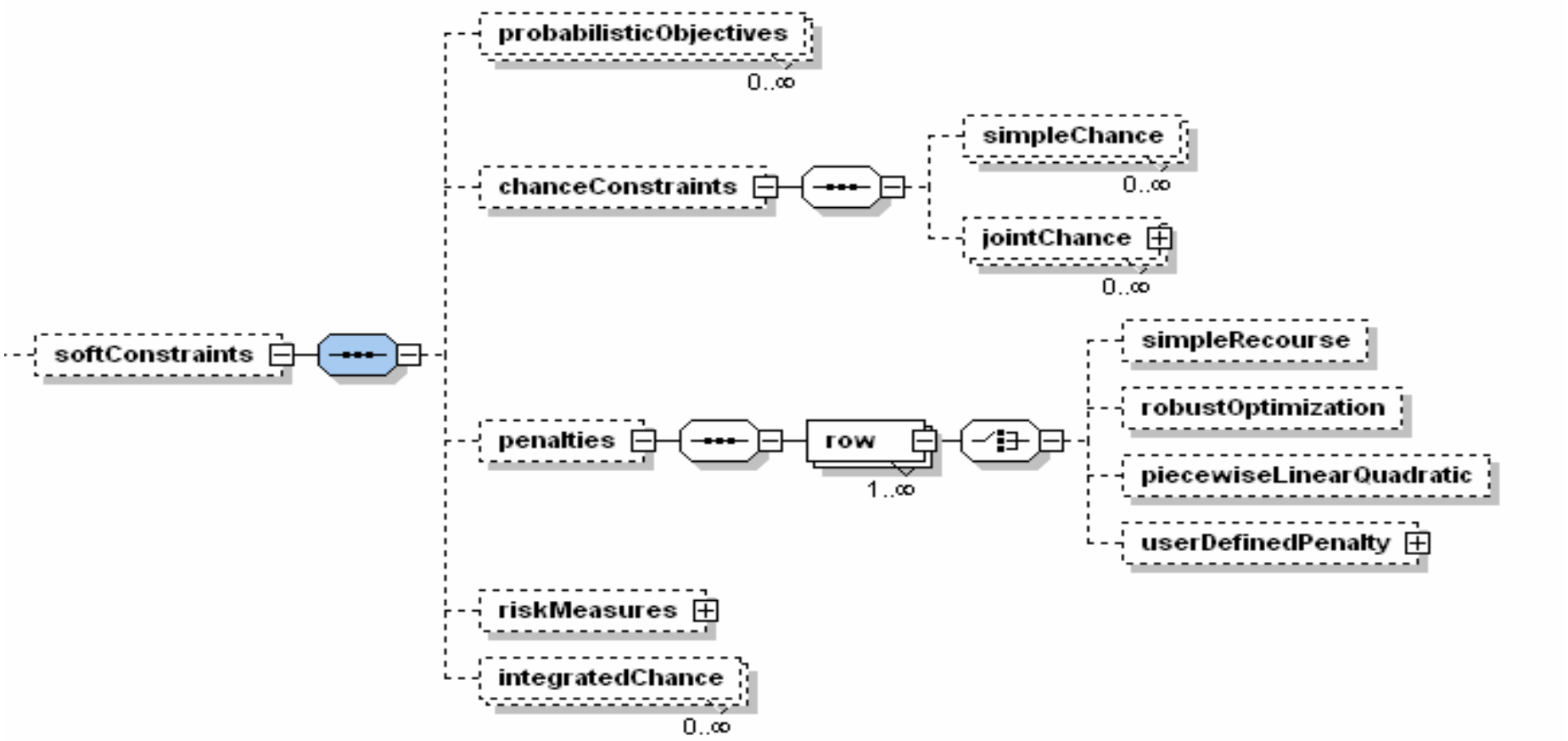

Transformations



OSiL – Linear transformation

```
<stochasticTransformation>
 <linearTransformation stage="0">
  <el name="dist1">
   <el rowIdx="3" colIdx="4"/>  <el rowIdx="3" colIdx="5"/>
  </el>
  <randomVariableList>
   <coefMatrix>
    <listMatrix>
     <start>    <el>0</el>    <el>1</el>    <el>2</el>    </start>
     <rowIdx>  <el>0</el>    <el>1</el>    </rowIdx>
     <value>    <el>1.0</el>  <el>1.0</el>  </value>
    </listMatrix>
   </coefMatrix>
  </randomVariableList>
 </linearTransformation>
 <linearTransformation stage="1">
 …
</stochasticTransformation>
```

Penalties and probabilistic constraints



Capabilities

- Arbitrary nonlinear expressions
- Arbitrary distributions
- Scenario trees
- Stochastic problem dimensions
- Simple recourse
- Soft constraints with arbitrary penalties
- Probabilistic constraints
- Arbitrary moment constraints

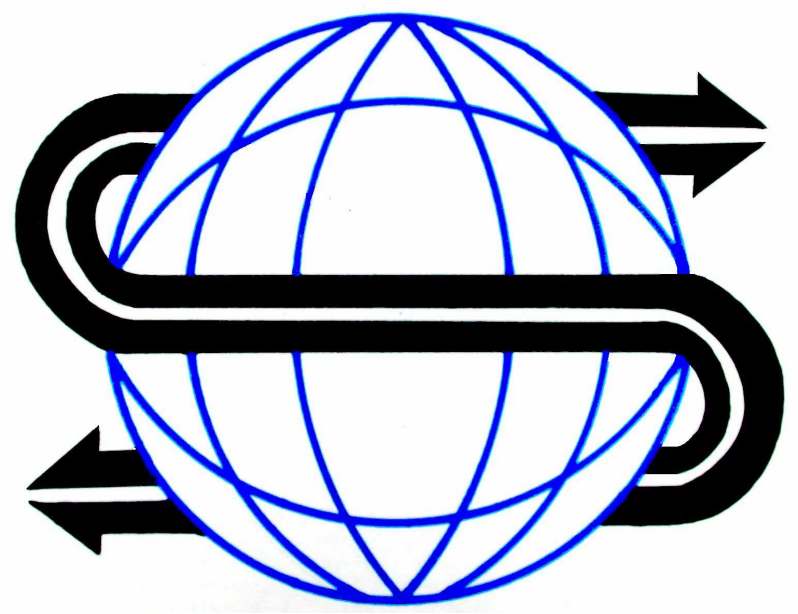
Nonlinear expression – $(x_0 - x_1^2)^2 + (1 - x_0)^2$

```
<plus>
  <power>
    <minus>
      <var coef="1.0" idx="0"/>
      <power>
        <var coef="1.0" idx="1"/>
        <number value="2"/>
      </power>
    </minus>
    <number value="2"/>
  </power>
  <power>
    <minus>
      <number value="1"/>
      <var coef="1.0" idx="1"/>
    </minus>
    <number value="2"/>
  </power>
</plus>
```


Further work

- Readers
- Internal data structures
- Solver interfaces
- Library of problems
- Buy-in

QUESTIONS?



http://www.optimizationservices.org

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