Extensions to an Optimization Services Instance Language

Robert Fourer, Jun Ma

Industrial Engineering & Management Sciences
McCormick School of Engineering and Applied Science
Northwestern University

{4er,maj}@iems.northwestern.edu

Kipp Martin

Graduate School of Business
University of Chicago

kmartin@gsb.uchicago.edu

IFORS 2005

Honolulu, Hawaii — Thursday, July 14, 2005 — RA-14.3

Abstract

- Optimization problems of interest today go beyond the traditional linear, integer, quadratic, and smooth nonlinear types. A language for problem instances must be extended accordingly. This presentation describes prospective extensions to OSiL, our proposed language standard, in such areas as combinatorial optimization and constraint programming, stochastic programming, and semidefinite and cone programming.

Why Extensions?

Better describe problem instances

- Describe broader variety
- Describe more concisely
- ~~Make more natural and understandable~~

More readily transform models and problems

- Modeling system → OSiL
- OSiL → Solver

Outline

Piecewise-linear terms

Logic & combinatorial constraints

Complementarity constraints

Stochastic programming

Cone & semidefinite programming

Piecewise-Linear Terms

Design considerations

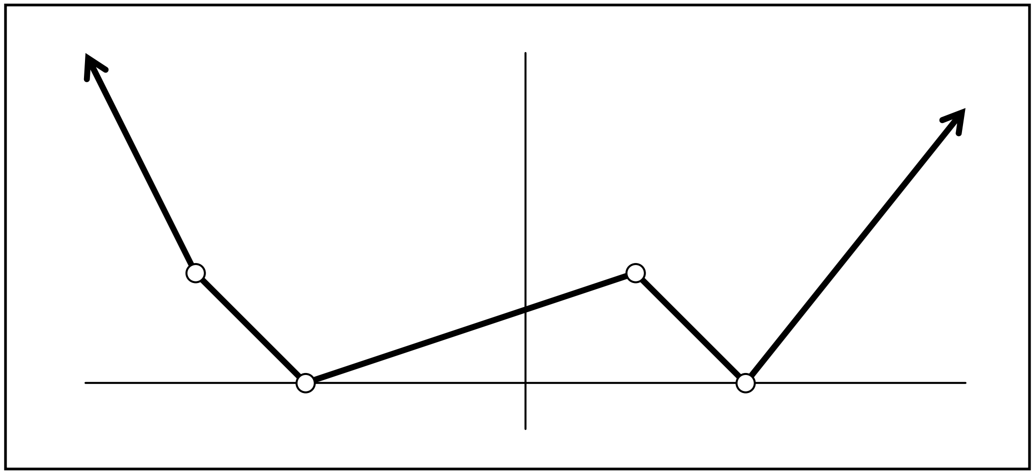
- ➢ Univariate function of a numerical expression
- ➢ Defined by alternating breakpoints and slopes
  - ∗ Start and end with slopes
  - ∗ Value at zero is zero unless overridden
- ➢ Ordering of pieces must be unambiguous

Tentative extensions

- ➢ Series of `<piece>` elements
  - ∗ one slope and one breakpoint attribute
  - ∗ last `<piece>` has no breakpoint
  - ∗ sorted by breakpoint value
- ➢ Optional `<level>` element gives value at zero
- ➢ A final element specifies the operand as an expression tree

Example

```
<piecewiseLinear>
   <piece slope="-2" breakpoint=‘‘-3"/>
   <piece slope="=1" breakpoint="-2"/>
   <piece slope="0.333333" breakpoint="1"/>
   <piece slope="-1" breakpoint="2"/>
   <piece slope="1.25"/>
   <level value=‘‘0.666667"/>
   <var idx=‘‘7">
</piecewiseLinear>
```



Logic and Combinatorial Constraints

Design considerations

- Expression types
- Constraint types
- New operators

Examples of tentative extensions

- Logic operators
- Counting operator
- "All different" operator
- Variable indexed by a variable

Expression Types

Numerical

- Value is a number

```
var Trans {ORIG, DEST} >= 0;
```

Logical

- Value is “true” or “false”

Object

- Value is a member of some set

```
var JobForSlot {SLOTS} in JOBS;
```

Set

- Value is a set of numbers or objects:

```
var MEMBERS {PROJECTS} within VOLUNTEERS;
```

Constraint Types

Range constraints

- $lowerBound \le numExpr \le upperBound$
- For one-sided constraint,
  $lowerBound = -\infty$ or $upperBound = +\infty$
- For equality, $lowerBound = upperBound$

Logic constraints

- *logicExpr*
- Logical
```
(Mk[i] = 0 and Mk[i] = 0) or Mk[i] + Mk[i] >= lbd
```
- Counting
```
atmost mxsrv {j in D} (sum {p in PRD} Tr[i,j,p] >= 10)
```
- Special-structure
```
alldiff {j in Jobs} (MachineForJob[j])
```

New Operators

Numerical-valued on constraints

- Counting

```
count {j in D} (sum {p in PRD} Tr[i,j,p] >= 10)
```

Logic-valued on constraints

- Logical

```
(Mk[1] = 0 and Mk[2] = 0) or (Mk[1] + Mk[2] >= 100)
```

- Counting

```
atmost mxsrv {j in D} (sum {p in PRD} Tr[i,j,p] >= 10)
```

Special-structure ("global")

- All-different

```
alldiff {j in Jobs} (MachineForJob[j])
```

- Distribution

```
numberof 3 in ({j in 1..nJobs} MachineForJob[j])
```

New Operators *(cont'd)*

Indexing

- Variables in subscripts of parameters or variables

```
param mCLI integer > 0;
param nLOC integer > 0;

param srvCost {1..mCLI, 1..nLOC} > 0;
param bdgCost > 0;

var Serve {1..mCLI} integer >= 1, <= nLOC;
var Open {1..nLOC} integer >= 0, <= 1;

minimize TotalCost:
   sum {i in 1..mCLI} srvCost[i,Serve[i]] +
   bdgCost * sum {j in 1..nLOC} Open[j];

subject to OpenDefn {i in 1..mCLI}:
   Open[Serve[i]] = 1;
```

New Operators *(cont'd)*

Indexing

- Variables constrained by subscripts

```
set ABLE within {1..mCLI, 1..nLOC};
param srvCost {ABLE} > 0;

.......

minimize TotalCost:
   sum {i in 1..mCLI} srvCost[i,Serve[i]] + ...
```

`...(i,Serve[i])` ***must be in*** **`ABLE`**

With set operands

- Set valued: union, intersection, difference
- Numerical valued: cardinality
- Logic valued: membership, containment
- Special-structure: all-disjoint

Logic Extensions to OSiL

Design

- Use same “nonlinear” expression tree
- Define new nodes to represent new operators

Implementation

- Extend API to give solvers
  access to constraint expressions

Example: Logic Operators

```
(Mk[i] = 0 and Mk[i] = 0) or Mk[i] + Mk[i] >= lbd
```

```
<or>
   <and>
      <eq>
         <var idx="23"/>
         <number value="0"/>
      </eq>
      <eq>
         <var idx="103"/>
         <number value="0"/>
      </eq>
   </and>
   <geq>
      <plus>
         <var idx="23"/>
         <var idx="103"/>
      </plus>
      <number value="150"/>
   </geq>
</or>
```

Example: “at most” operator

```
atmost mxsrv {j in D} (sum {p in PRD} Tr[i,j,p] >= lim[j])
```

```
<atMost>
   <number value="2"/>
   <geq>
      <sum>
         <var idx=" 20"/>
         <var idx=" 21"/>
         <var idx=" 22"/>
      </sum>
      <number value="10"/>
   </geq>
   <geq>
      <sum>
         <var idx=‘‘30‘‘/>
         <var idx=‘‘31"/>
         <var idx=‘‘32"/>
      </sum>
      <number value=‘‘27"/>
   </geq>
   <geq>
   ...
</atMost>
```

Examples *(cont'd)*

`alldiff {j in Jobs} (MachineForJob[j])`

```
<alldiff>
   <var idx="27"/>
   <var idx="37"/>
   <var idx="47"/>
</alldiff>
```

`Open[Serve[7]]` where `mCLI = 40, nLOC = 15`

- `Serve corresponds to Var[0], ..., Var[39]`
- `Open corresponds to Var[40], ..., Var[54]`

```
<var>
   <plus>
      <number value="39"/>
      <var idx="6"/>
   </plus>
</var>
```

Complementarity

Definition

- Two inequalities must hold . . .
- At least one of them with equality

Applications

- Equilibrium problems in economics and engineering
- Optimality conditions for nonlinear programs,
  bi-level linear programs, etc.

Forms

- “Square” systems of complementarity conditions
  - # of variables =
    # of complementarity constraints + # of equality constraints
- Mathematical programs
  with complementarity constraints (MPCCs)

Classical Complementarity

LP with nonnegative variables

- Complementary slackness conditions

```
PrimalConstr {i in I}:
  sum {j in J} a[i,j] * X[j] = b[i];

PrimalBounds {j in J}: X[j] >= 0;

DualConstr {j in J}:
  sum {i in I} Y[i] * a[i,j] + Z[j] = c[j];

DualBounds {i in I}: Z[i] >= 0;

Complementarity {j in J}:
  X[j] = 0 or Z[j] = 0;
```

- Multiplicative alternative

```
Complementarity {j in J}:
  X[j] * Z[j] = 0;
```

Mixed Complementarity

LP with bounded variables

- Complementary slackness conditions

```
PrimalConstr {i in I}: sum {j in J} a[i,j] * X[j] = b[i];
PrimalBounds {j in J}: l[j] <= X[j] <= u[j];

DualConstr {j in J}:
  sum {i in I} Y[i] * a[i,j] + Z[j] = c[j];

Complementarity {j in J}:
  X[j] = l[j] implies Z[j] >= 0 and
  X[j] = u[j] implies Z[j] <= 0 and
  l[j] < X[j] < u[j] implies Z[j] = 0;
```

- Variational inequality alternative

```
Complementarity {j in J}:
  forall {Y[j] in interval[l[j],u[j]]}
    (Y[j] - X[j]) * Z[j] >= 0;
```

New complements Operator

LP with nonnegative variables

```
PrimalConstr {i in I}:
  sum {j in J} a[i,j] * X[j] = b[i];

DualConstr {j in J}:
  sum {i in I} Y[i] * a[i,j] + Z[j] = c[j];

Complementarity {j in J}:
  X[j] >= 0 complements Z[j] >= 0;
```

LP with bounded variables

```
PrimalConstr {i in I}:
  sum {j in J} a[i,j] * X[j] = b[i];

DualConstr {j in J}:
  sum {i in I} Y[i] * a[i,j] + Z[j] = c[j];

Complementarity {j in J}:
  l[j] <= X[j] <= u[j] complements Z[j];
```

. . . without Auxiliary Variable Z[j]

LP with nonnegative variables

```
PrimalConstr {i in I}:
  sum {j in J} a[i,j] * X[j] = b[i];

Complementarity {j in J}:
  X[j] >= 0 complements
    sum {i in I} Y[i] * a[i,j] <= c[j];
```

LP with bounded variables

```
PrimalConstr {i in I}:
  sum {j in J} a[i,j] * X[j] = b[i];

Complementarity {j in J}:
  l[j] <= X[j] <= u[j] complements
    c[j] - sum {i in I} Y[i] * a[i,j];
```

Nonlinear

Price-dependent demands

```
var Price {i in PROD};
var Level {j in ACT};

subject to Pri_Compl {i in PROD}:
   Price[i] >= 0 complements
      sum {j in ACT} io[i,j] * Level[j]
         >= demzero[i] - demrate[i] * Price[i];

subject to Lev_Compl {j in ACT}:
   Level[j] >= 0 complements
      sum {i in PROD} Price[i] * io[i,j] <= cost[j];
```

. . . not obviously an optimality condition for an optimization problem

From Applications

Prices of coal shipments

```
subject to delct {cr in creg, u in users}:

  0 <= ct[cr,u] complements

    ctcost[cr,u] + cv[cr] >= p["C",u];
```

Height of membrane

```
subject to dv {i in 1..M, j in 1..N}:

  lb[i,j] <= v[i,j] <= ub[i,j] complements

     (dy/dx) * (2*v[i,j] - v[i+1,j] - v[i-1,j])
   + (dx/dy) * (2*v[i,j] - v[i,j+1] - v[i,j-1])
   - c * dx * dy ;
```

. . . more at Complementarity Problem Net

`http://www.cs.wisc.edu/cpnet/`

Operands: Always Two Inequalities

Two single inequalities

- *single-ineq1* `complements` *single-ineq2*
  - ∗ Both inequalities must hold,
  - ∗ at least one at equality

One double inequality

- *double-ineq* `complements` *expr*
  expr `complements` *double-ineq*
  - ∗ The double-inequality must hold, and
  - ∗ if at lower limit then $expr \geq 0$,
    if at upper limit then $expr \leq 0$,
    if between limits then $expr = 0$

Complementarity Extensions to OSiL

Design

- Introduce new **`<complements>`** element
  to expression tree
- Require two child nodes

Implementation

- Check for "two inequalities" requirement
  after the validation phase

Example

- `p[1] >= 0 complements`
```
400*h[0]^3*p[1]/exp(1.416*p[1]) -
400*h[1]^3*(p[2]-p[1])/exp(1.416*(p[2]+p[1])) +
121.14*h[1] - 121.14*h[0] >= 0;
```

```
<complements>
  <geq>
    <var idx="25"/>
    <number value="0"/>
  </geq>
  <geq>
    <sum>
      <times>
        <number value="400"/>
        <times>
          <power>
            <var idx="47"/>
            <number value="3"/>
          </power> ...
    </sum>
    <number value="0">
  </geq>
</complements>
```

Example *(more completed)*

```
<complements>
  <geq>
    <var idx="25"/>
    <number value="0"/>
  </geq>
  <geq>
    <sum>
      <times>
        <number value="400"/>
        <times>
          <power>
            <var idx="47"/>
            <number value="3"/>
          </power>
          ...
        </times>
      </times>
      ...
      <var idx="47" coef="-121.14"/>
    </sum>
    <number value="0">
  </geq>
</complements>
```

Stochastic Programming

Core optimization problem

Stage information

Stochastic information

What about SMPS Format?

Shares drawbacks of the MPS format for LPs, MIPs

- Limited precision
- Limited name length
- Expensive to process
- Restricted to linear problems
- Not entirely standard

Yet doesn't cover all problems of interest

- *See* extensions at `sba.management.dal.ca/profs/hgassmann/smps2.htm`

OSiL Stochastic Extensions

XML-based format

- ***Core*** LP or MIP described using OSiL
- Multi-stage partitioning using `<stages>` element
- Stochastic characteristics using `<stochastic>` element

Example (Birge & Louveaux, *Intro to Stoch Prog*)

General information for core problem

```
<problemDescription>
  <source>FinPlan_JohnBirge</source>
  <maxOrMin>max</maxOrMin>
  <objConstant>0.</objConstant>
  <numberObjectives>1</numberObjectives>
  <numberConstraints>4</numberConstraints>
  <numberVariables>8</numberVariables>
</problemDescription>
```

Example *(cont'd)*

Constraint & variable data for core problem

```
<problemData>
  <constraints>
    <con name="p1" lb="55" ub="55" />
    <con name="p2" lb="0" ub="0" />
    <con name="p3" lb="0" ub="0" />
    <con name="p4" lb="80" ub="80" />
  </constraints>
  <variables>
    <var name="x11" />
    <var name="x21" />
    <var name="x12" />
    <var name="x22" />
    <var name="x13" />
    <var name="x23" />
    <var objCoef="1" name="y" />
    <var objCoef="-4" name="w" />
  </variables>
```

Example *(cont'd)*

Coefficient column-start positions for core problem

```
<coefMatrix>
  <start>
    <el>0</el>
    <el>2</el>
    <el>4</el>
    <el>6</el>
    <el>8</el>
    <el>10</el>
    <el>12</el>
    <el>13</el>
  </start>
```

Example *(cont'd)*

Coefficients for core problem

```
<rowIdx>
  <el>0</el>
  <el>1</el>
  <el>0</el>
  <el>1</el>
  <el>1</el>
  <el>2</el>
  <el>1</el>
  <el>2</el>
  <el>2</el>
  <el>3</el>
  <el>2</el>
  <el>3</el>
  <el>3</el>
  <el>3</el>
</rowIdx>
```

```
  <value>
    <el>1</el>
    <el>-1.25</el>
    <el>1</el>
    <el>-1.14</el>
    <el>1</el>
    <el>-1.25</el>
    <el>1</el>
    <el>-1.14</el>
    <el>1</el>
    <el>-1.25</el>
    <el>1</el>
    <el>-1.14</el>
    <el>-1</el>
    <el>1</el>
  </value>
</coefMatrix>
```

OSiL `<stages>` Element

Partition variables & constraints by stage number

- Required attribute gives number of stages

`<implicitOrder>`

- When already ordered by stage
- Just say where each stage begins

`<explicitOrder>`

- Specify a stage number for each variable and constraint

Example *(cont'd)*

Stage information

```
<stages number="4">
  <implicitOrder>
    <el startRowIdx="1" startColIdx="1" />
    <el startRowIdx="2" startColIdx="3" />
    <el startRowIdx="3" startColIdx="5" />
    <el startRowIdx="4" startColIdx="7" />
  </implicitOrder>
</stages>
```

OSiL `<stochastic>` Element

<explicitScenario>

- Discrete distributions
- Discrete approximations to distributions

<implicitScenario>

- Continuous distributions

<penalties>

- Simple recourse

<riskMeasures>

- Chance constraints
- Probabilistic objectives

Alternatives for Explicit Scenarios

`<scenarioPath>`

- Every child represents a scenario as a ***path***
  - from the root of the scenario tree
  - to one of its leaves
- First child is the ***root scenario***
  - defined by the core problem
- Each subsequent child branches
  - directly from the root
  - or indirectly from some previous branch

Every scenario has a parent

- Only differences from parent are specified

Example *(cont'd)*

Explicit scenarios by path

```
<stochastic>
  <explicitScenario>
     <scenarioPaths>
       <rootScenario name="sc1" prob="0.125" base="none" stage="1">
         <el rowIdx="2" colIdx="1">1.25</el>
         <el rowIdx="2" colIdx="2">1.14</el>
         <el rowIdx="3" colIdx="3">1.25</el>
         <el rowIdx="3" colIdx="4">1.14</el>
         <el rowIdx="4" colIdx="5">1.25</el>
         <el rowIdx="5" colIdx="5">1.14</el>
         <bound varIdx="2" type="lower"/>
       </rootScenario>
       <scenario name="sc2" prob="0.125" parent="1" stage="4">
         <el rowIdx="4" colIdx="5">1.06</el>
         <el rowIdx="5" colIdx="5">1.12</el>
       </scenario>
       <scenario name="sc3" prob="0.125" parent="1" stage="3">
         <el rowIdx="3" colIdx="3">1.06</el>
         <el rowIdx="3" colIdx="4">1.12</el>
         <el rowIdx="4" colIdx="5">1.25</el> ...
```

Alternatives for Explicit Scenarios

<scenarioTree>

- Every child represents a ***node*** of the scenario tree
  - by means of an `<snode>` element
- First `<snode>` corresponds to the ***root node***
- Every `<snode>` may have `<snode>` children
  - defining branches of the tree

Every <snode> specifies the problem at its stage

- By listing differences from its parent, ***or***
- By specifying a single-stage OSiL problem

Example *(cont'd)*

Explicit scenarios by tree node

```
<stochastic>
  <explicitScenario>
    <scenarioTree>
      <sNode prob="1" base="coreProgram">
        <changes>
          <el rowIdx="2" colIdx="1">1.06</el>
          <el rowIdx="2" colIdx="2">1.12</el>
        </changes>
        <sNode prob="0.5" base="coreProgram">
          <sNode prob="0.5" base="coreProgram">
            <sNode prob="0.5" base="coreProgram"/>
            <sNode prob="0.5" base="firstSibling">
              <changes>
                <el rowIdx="4" colIdx="5">1.06</el>
                <el rowIdx="5" colIdx="5">1.12</el>
              </changes>
            </sNode>
          </sNode>
          <sNode prob="0.5" base="firstSibling">
            <changes>
              <el rowIdx="3" colIdx="3">1.06</el> ...
```

Alternatives for Implicit Scenarios

<distributions>

- Built-in univariate and multivariate
- User-defined

<stochasticElements>

- Series of `<elementGroup>` children
- Each `<elementGroup>` specifies a stochastic process

$$Y_t = \sum_{i=1}^{p} M_i Y_{t-i} + \sum_{j=1}^{q} N_j v_{t-j} + c_t$$

- ∗ matrices M_i, N_j
- ∗ uncorrelated, identically distributed random vectors v_{t-j}
- ∗ constant c_t

. . . *an* ARMA(*p*, *q*) *process*

<penalties> **Element**

Specifies penalties for violating constraints

- `<simpleRecourse>` :
  linear shortage and surplus penalties
- `<robustOptimization>` : quadratic penalties
- `<piecewiseLinearQuadratic>`
- `<userDefinedPenalty>` : shortage and surplus
  specified like other user-defined functions

. . . ***separate for each constraint***

`<riskMeasures>` Element

Each child may take any of three types

- `<simpleChance>`
- `<jointChance>`
- `<integratedChance>` (*see* . . .)

One (simple) or more (joint) `rowIdx` *attributes*

- `rowIdx` ≥ 0 implies chance constraint
  - probability that the constraint is satisfied
- `rowIdx` < 0 implies probabilistic objective
  - minimize or maximize the *probability* that the objective is ≥ or ≤ a constant

Cone and Semidefinite Programming

Design considerations

- Generalizations of $x \geq 0$
- Small number of cone types

Tentative extensions

- An element for each cone type
- Child elements and/or attributes
  indicate the variables involved

Cone Types

Second-order

- Quadratic cone

$$x_1^2 \geq \sum_{j=2}^{n} x_j^2$$

- Rotated quadratic cone

$$2x_1x_2 \geq \sum_{j=3}^{n} x_j^2$$

Semidefinite

- Symmetric matrix X of variables is positive semi-definite

Example

Constraint & variable data for core problem

```
<cones>
   <quadraticCone>
      <el>1</el>
      <el>3</el>
   </quadraticCone>
   <quadraticCone>
      <el>2</el>
      <el mult="3" incr="1">4</el>
   </quadraticCone>
   <rotatedQuadraticCone startIndex="7" endIndex="9"/>
</cones>
```