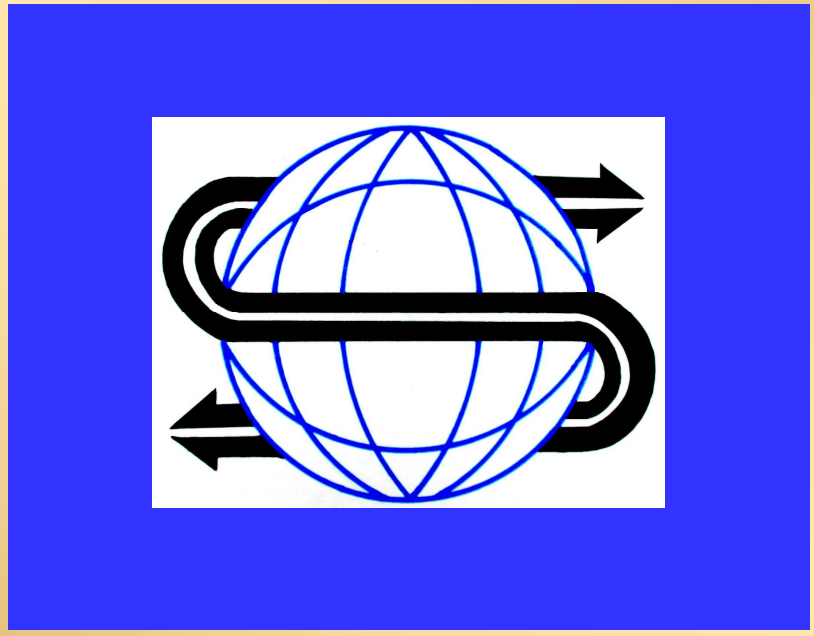

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OSiL: An XML-based schema for stochastic programs

H.I. Gassmann, R. Fourer, J. Ma, R.K. Martin

SP XI Vienna, August 2007

Outline

- Motivation and review
- A four-stage investment problem
- OSiL format
- Conclusions and future work

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Why a standard?

- Benchmarking
- Archiving
- Algorithm development
- Distributed computing
- Sharing of problem instances

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Why XML?

- Existing parsers to check syntax
- Easy to generate automatically
- Tree structure naturally mirrors expression trees for nonlinear functions
- Arbitrary precision and name space
- Automatic attribute checking (e.g., nonnegativity)
- Querying capabilities via XQuery
- Encryption standards being developed
- Easy integration into broader IT infrastructure

Stochastic programs

$$
\begin{array}{lll}
\min & f_0(x_0) + f_1(x_0, x_1) + \ldots + f_T(x_0, x_1, \ldots, x_T) & \\
\text{“ s.t.„} & G_0(x_0) & \sim b_0 \\
 & R_1(x_0) & \Delta\ r_1 \\
 & G_1(x_0, x_1) & \Delta\ b_1 \\
 & \vdots & \vdots \\
 & G_T(x_0, x_1, \ldots, x_T) & \Delta\ b_T
\end{array}
$$

$$l_0 \le x_0 \le u_0$$

$$l_t \le x_t \le u_t, t = 1, \ldots, T$$

Any data item with nonzero subscript may be random (including dimensions where mathematically sensible)

~ stands for arbitrary relations ($\le$, =, $\ge$)

Δ means ~ with probability 1

or with probability at least β

or with expected violation at most v

or ...

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Problem classes and time domain

- Single-stage problems
  - Mean-variance problems (Markowitz)
  - Robust optimization
  - Chance-constrained problems

  - Reformulated and solved as deterministic nonlinear problems

Problem classes and time domain (cont'd)

- Two-stage problems with recourse
  - Solved by
    - Deterministic equivalent methods
    - Benders decomposition
    - Stochastic quasigradient methods
    - Stochastic decomposition (Higle and Sen)
    - Monte Carlo sampling (Shapiro and Homem-de-Mello)
    - Regression approximation (Deák)
  - Distributions
    - Known
    - Approximated (by scenario generation)
    - Partially known (moments, distribution type, support, etc.)

Problem classes and time domain (cont'd)

- Multi-stage recourse models
  - Deterministic equivalent
  - Nested Benders decomposition
  - Progressive hedging
  - Multistage stochastic decomposition
  - Probabilistic constraints and risk measures can be added as "linking constraints"

Problem classes and time domain (cont'd)

- Horizon problems (Grinold, Sethi)
- Markov reward processes
- Continuous time problems

Example (Birge)

$$\max \sum_{s=1}^{S} p_s (w_s - \beta u_s)$$

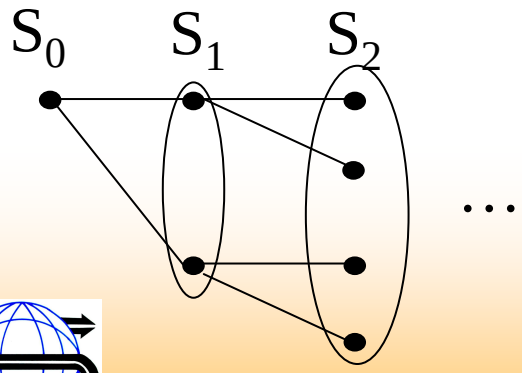
$$\text{s.t.} \sum_{i=1}^{I} x_{0i} = B$$

$$\sum_{i=1}^{I} \alpha_{0is} x_{0i} - \sum_{i=1}^{I} x_{1is} = 0, s \in S_1$$

$$\sum_{i=1}^{I} \alpha_{t-1,i,s} x_{t-1,i,a(s)} - \sum_{i=1}^{I} x_{tis} = 0, s \in S_t, t = 2, \ldots, T-1$$

$$\sum_{i=1}^{I} \alpha_{T-1,i,s} x_{T-1,i,a(s)} + u_s - w_s = R, s \in S_T$$

$$x_{tis}, u_s, w_s \geq 0$$



$I = 2,\ T = 3,\ B = 55,\ R = 80,$

$\alpha_{t1} = \{1.25, 1.06\},$

$\alpha_{t2} = \{1.14, 1.12\}$

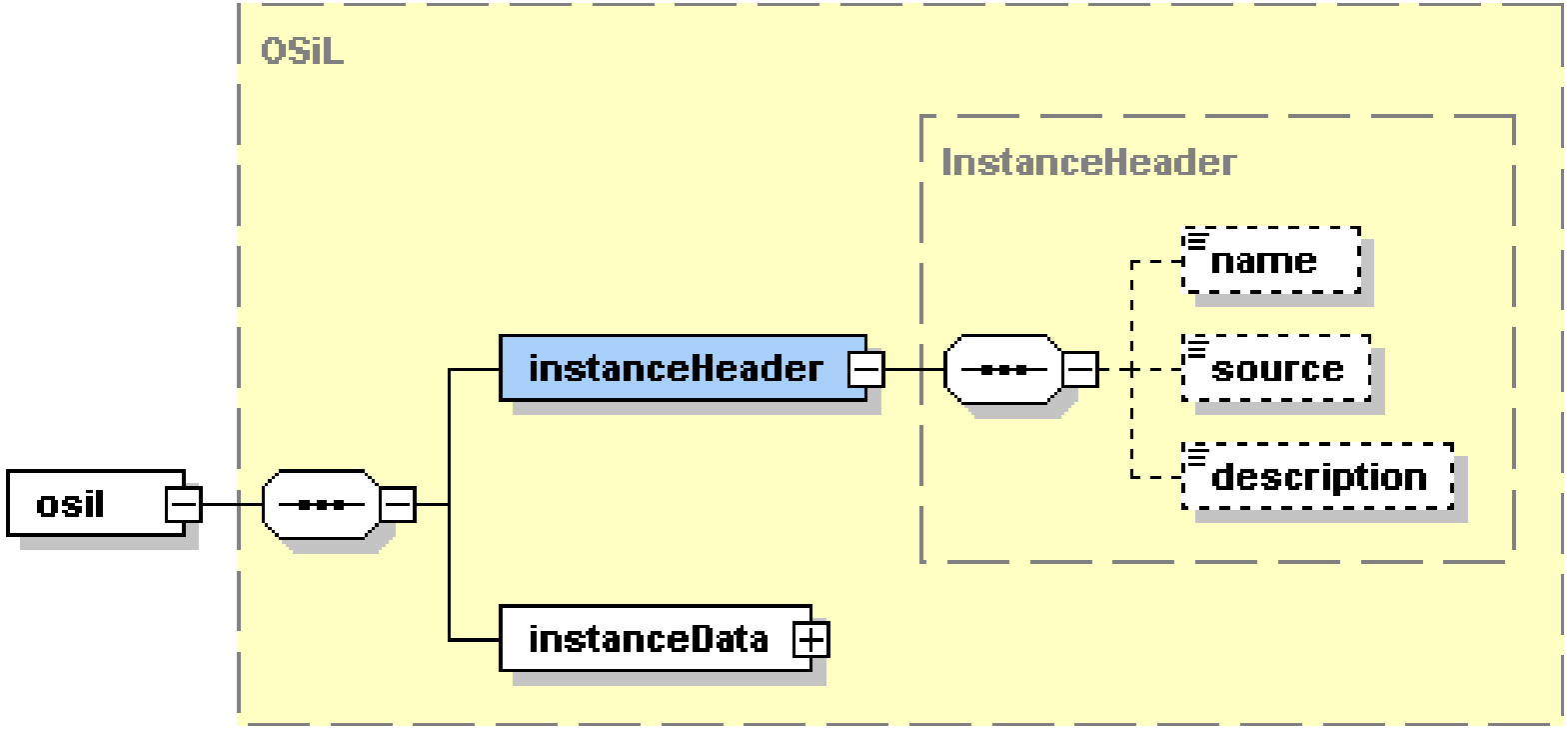
Markup languages

- Intersperse text (data) and information about it (formatting, etc.)
- Examples
  - TeX (extensible through user \def)
  - HTML
  - VRML
  - XML

OSiL Schema

- Written in XML
- Very flexible
- Intended to handle as many types of mathematical programs as possible
  - Linear and integer
  - Nonlinear
  - Stochastic
  - …

OSiL Schema – Header information



Header information – Example

```
<?xmlversion="1.0"encoding="UTF8"?>
  <osil xmlns="os.optimizationservices.org"
    xmlns:xsi=http://www.w3.org/2001/XMLSchemainstance
    xsi:schemaLocation="OSiL.xsd">
   <instanceHeader>
    <name>FinancialPlan_JohnBirge</name>
     <source>
        Birge and Louveaux, Stochastic Programming
     </source>
     <description>
        Three-stage stochastic investment problem
     </description>
   </instanceHeader >
   <instanceData>
     ...
   </instanceData>
  </osil>
```

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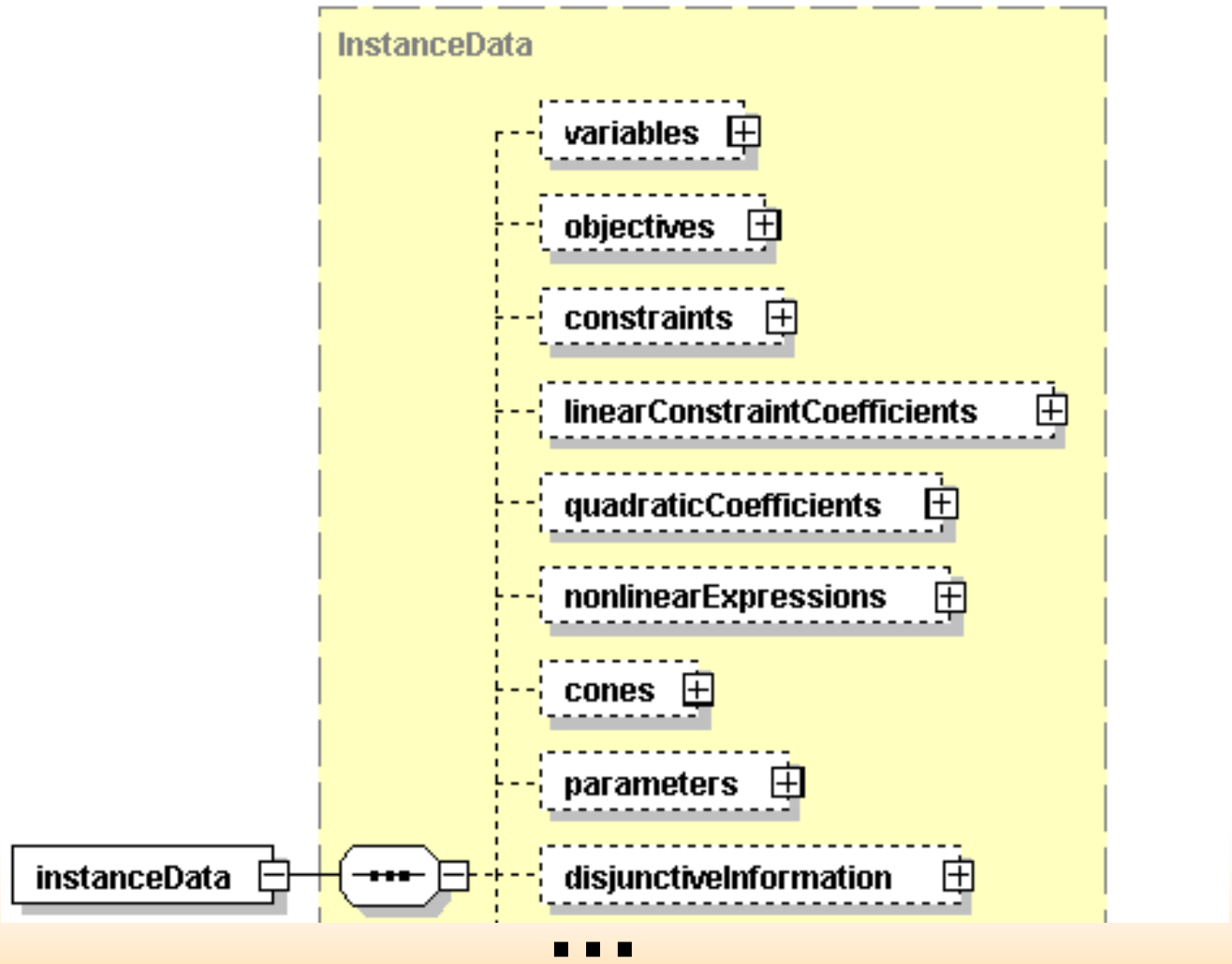
OSiL Schema – Deterministic data



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Instance data – Variables, objectives, constraints

```
<variables numberOfVariables="8">
   <var name="invest01" type="C" lb="0.0"/>
   <var name="invest02"/>
   <var name="invest11"/>
   <var name="invest12"/>
   <var name="invest21"/>
   <var name="invest22"/>
   <var name="w"/>
   <var name="u"/>
</variables>
<objectives numberOfObjectives="1">
   <obj maxOrMin="max" numberOfObjCoef= "2" lb="0.0">
      <coef idx="6"/>1.</coef>
      <coef idx="7"/>-4.</coef>
   </obj>
</objectives>
<constraints numberOfConstraints="4">
   <con name="budget0" lb="55" ub="55"/>
   <con name="budget1" lb="0" ub="0"/>
   <con name="budget2" lb="0" ub="0"/>
   <con name="budget3" lb="80" ub="80"/>
</constraints>
```

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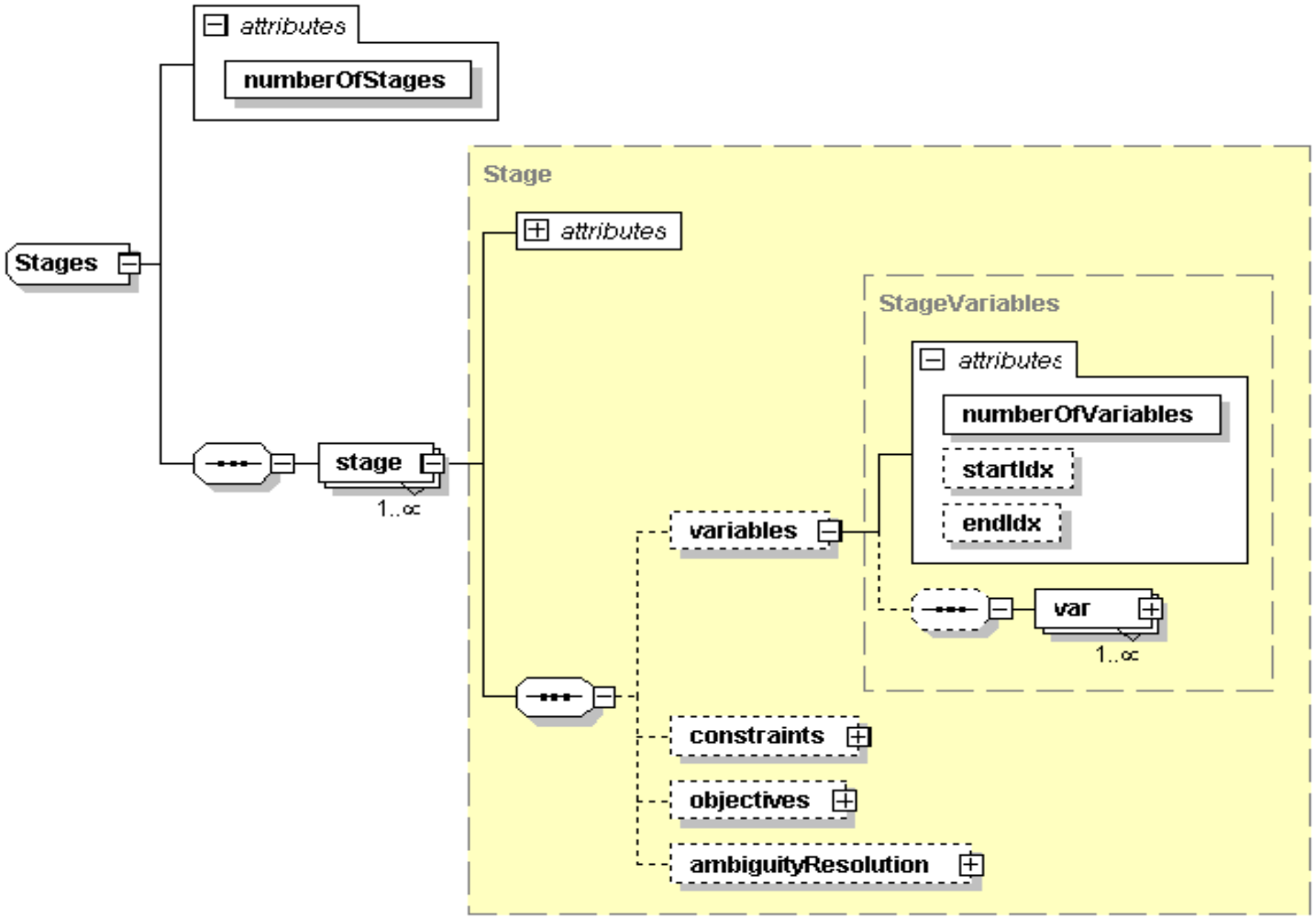
Instance data – Core matrix (sparse matrix form)

```
<linearConstraintCoefficients
        numberOfValues="14">
  <start>
    <el>0</el>
    <el>2</el>
    <el>4</el>
    <el>6</el>
    <el>8</el>
    <el>10</el>
    <el>12</el>
    <el>13</el>
    <el>14</el>
  </start>
```

```
  <rowIdx>
    <el>0</el>
    <el>1</el>
    <el>0</el>
    <el>1</el>
    <el>1</el>
    <el>2</el>
    <el>1</el>
    <el>2</el>
    <el>2</el>
    <el>3</el>
    <el>2</el>
    <el>3</el>
    <el>3</el>
    <el>3</el>
  </rowIdx>
```

```
  <value>
    <el>1</el>
    <el>1.25</el>
    <el>1</el>
    <el>1.14</el>
    <el>1</el>
    <el>1.25</el>
    <el>1</el>
    <el>1.14</el>
    <el>1</el>
    <el>1.25</el>
    <el>1</el>
    <el>1.14</el>
    <el>1</el>
    <el>-1</el>
  </value>
```

OSiL Schema – Dynamic structure



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Dynamic information – Example

```
<stages numberOfStages="4">
   <stage>
      <variables numberOfVariables="2" startIdx="0" endIdx="1"/>
      <constraints numberOfConstraints="1" startIdx="0"/>
   </stage>
   <stage>
      <variables numberOfVariables="2" startIdx="2" endIdx="3"/>
      <constraints numberOfConstraints="1" startIdx="1"/>
   </stage>
   <stage>
      <variables numberOfVariables="2" startIdx="4" endIdx="5"/>
      <constraints numberOfConstraints="1" startIdx="2"/>
   </stage>
   <stage>
      <variables numberOfVariables="2">
         <var idx="6">     <var idx="7">
      </variables>
      <constraints numberOfConstraints="1" startIdx="3"/>
   </stage>
</stages>
```

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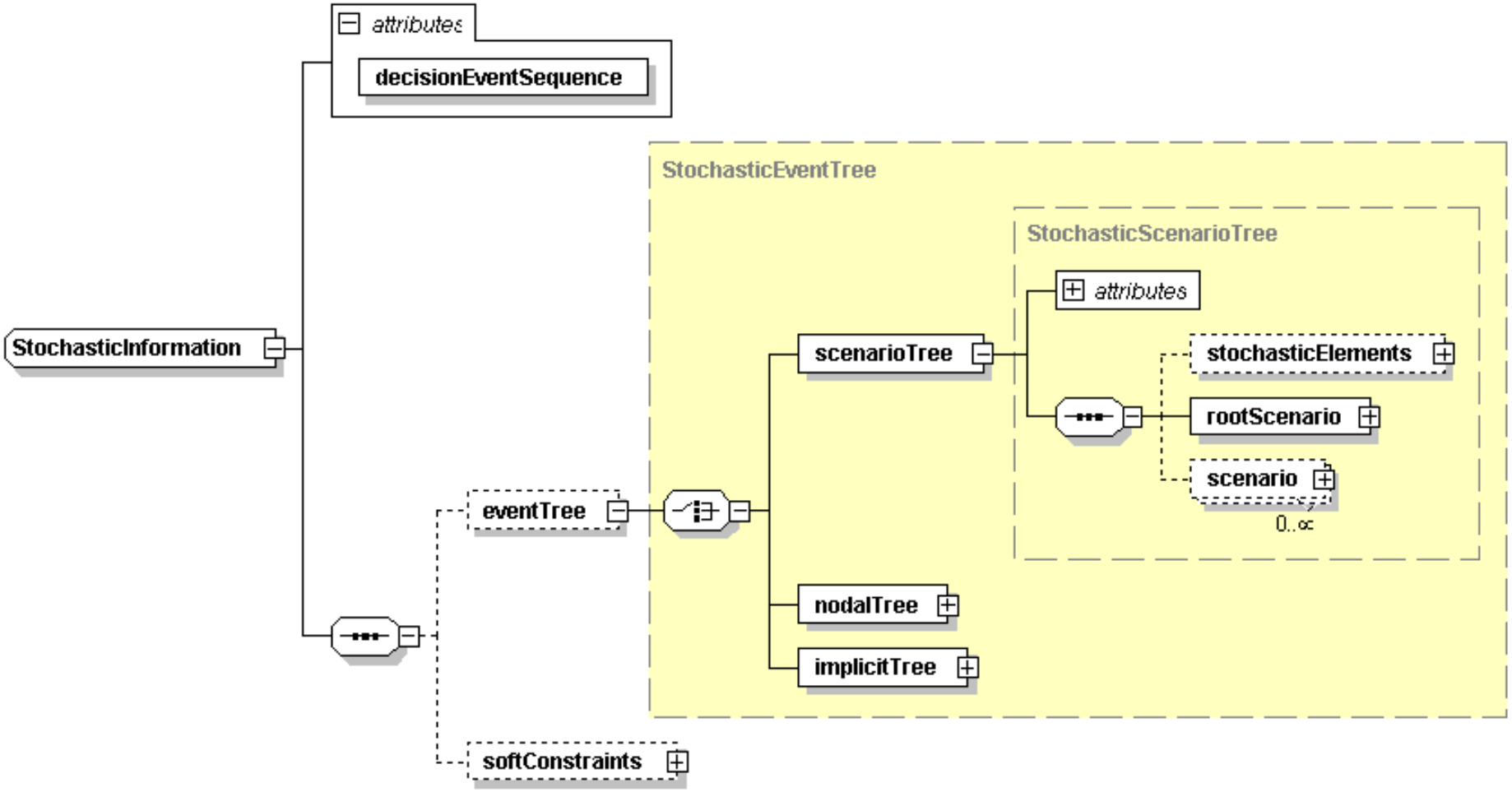
Explicit and implicit event trees



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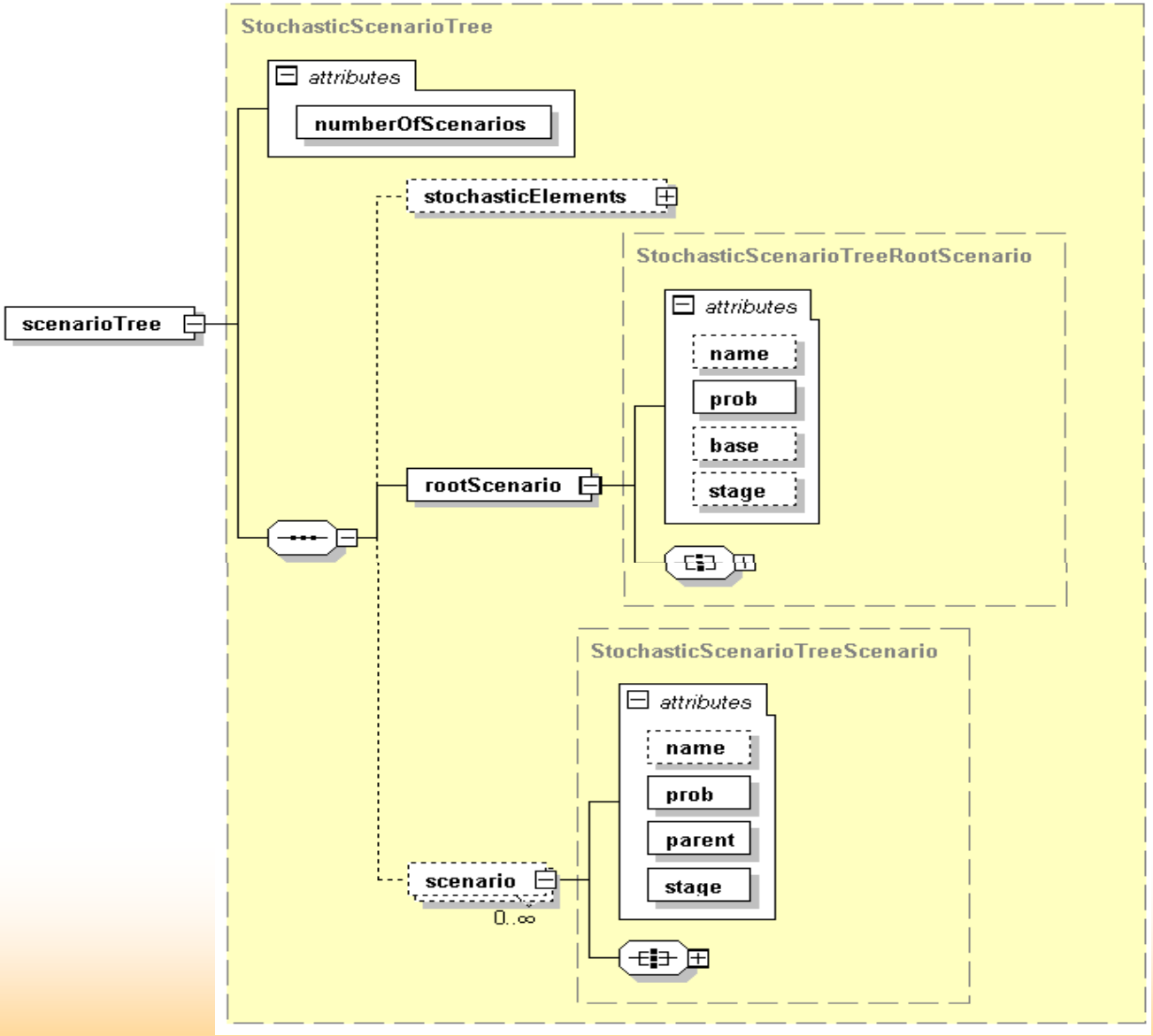
OSiL Schema – Scenario trees



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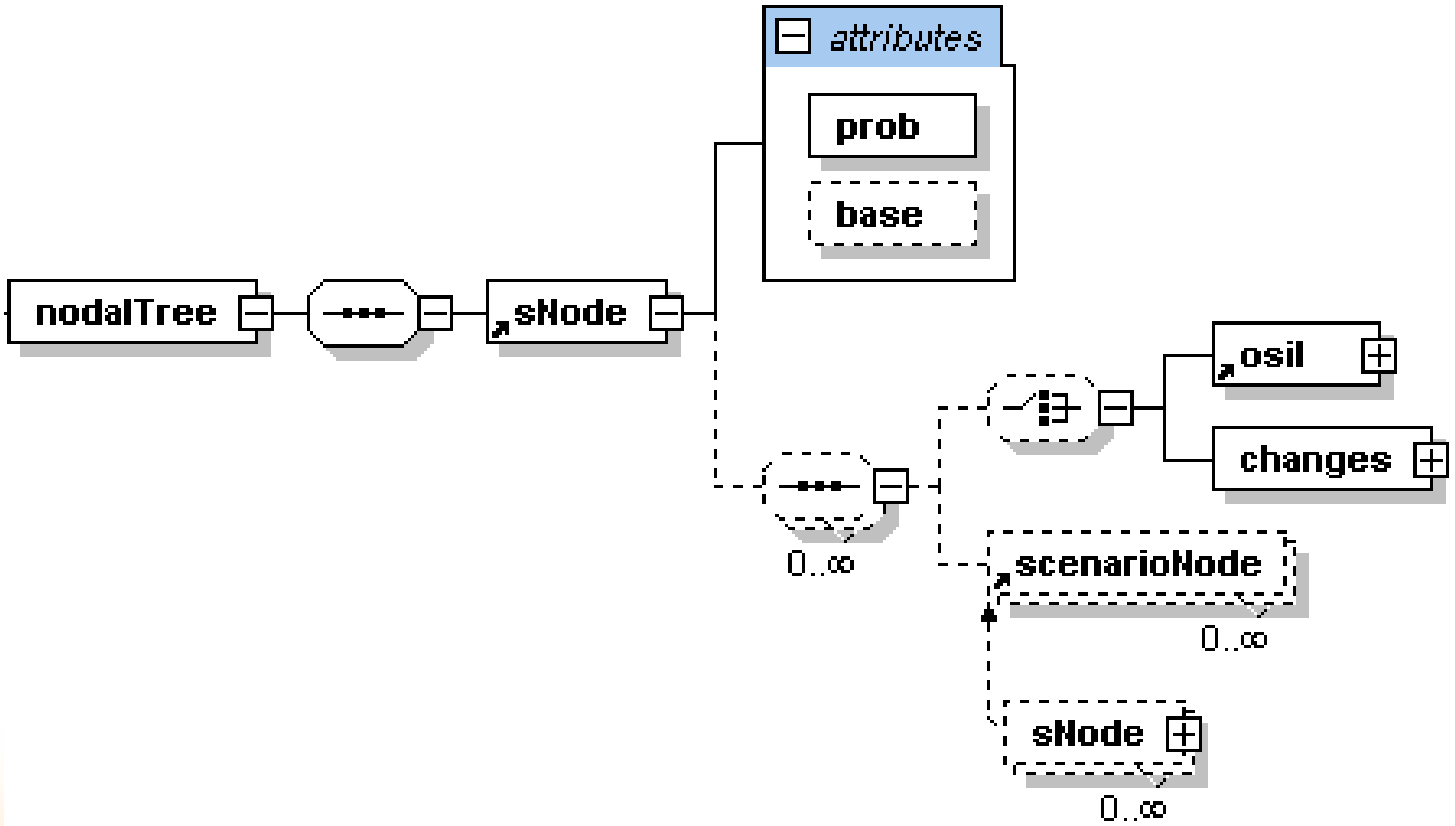
Scenario tree – Example

```
<stochasticInformation
        decisionEvenSequence="DecisionAfterEvent">
   <eventTree>
      <scenarioTree numberOfScenarios="8">
         <rootScenario prob="0.125" stage="0"/>
         <scenario prob="0.125" stage="3" parent="0">
            <linearConstraintCoefficients>
               <el rowIdx="3" colIdx="4">1.06</el>
               <el rowIdx="3" colIdx="5">1.12</el>
            </linearConstraintCoefficients>
         </scenario>
         <scenario prob="0.125" stage="2" parent="0">
          ...
```

Node-by-node representation for stochastic problem dimensions



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Node-by-node – Example

```
<stochasticInformation
        decisionEvenSequence="DecisionAfterEvent">
   <eventTree >
      <nodalTree>
         <sNode prob="1" base="coreProgram">
            <sNode prob="0.5" base="coreProgram">
               <sNode prob="0.5" base="coreProgram">
                  <sNode prob="0.5" base="coreProgram"/>
                  <sNode prob="0.5" base="firstSibling">
                     <changes>
                        <el rowIdx="3" colIdx="4">1.06</el>
                        <el rowIdx="3" colIdx="5">1.12</el>
                     </changes>
                  </sNode>
               </sNode>
            ...
```

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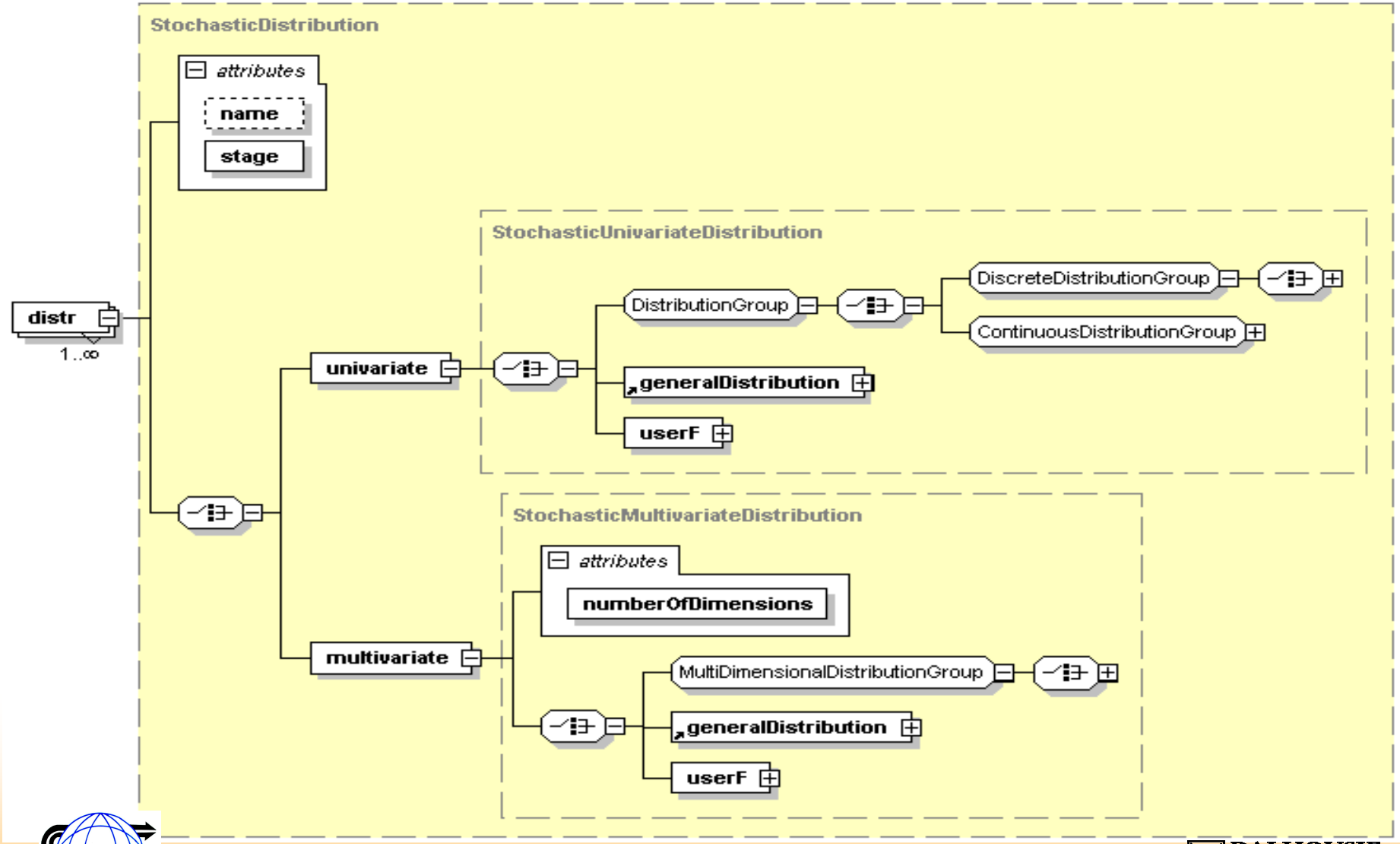
Distributions (implicit tree)



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Discrete random vector

```
<distributions>
   <multivariate>
      <distr stage="1">
         <multiDimensionalDistributionGroup>
            <multivariateDiscrete>
               <scenario>
                  <prob>0.5</prob>
                  <el>1.25</el>
                  <el>1.14</el>
               </scenario>
               <scenario>
                  <prob>0.5</prob>
                  <el>1.06</el>
                  <el>1.12</el>
               </scenario>
            </multivariateDiscrete>
         </multiDimensionalDistributionGroup>
      </distr>
      ...
   </multivariate>
</distributions>
```

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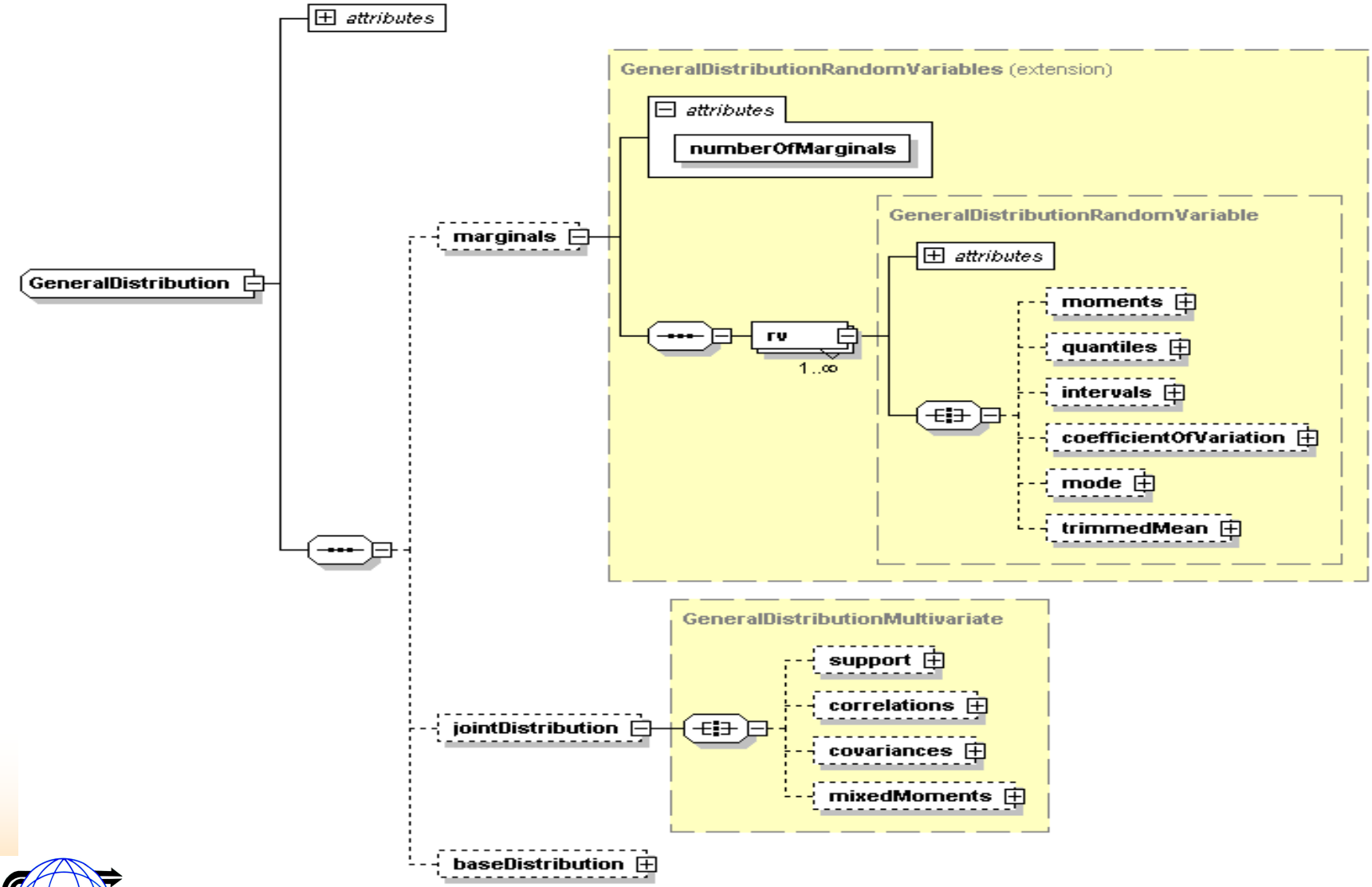
General distribution (incomplete information)



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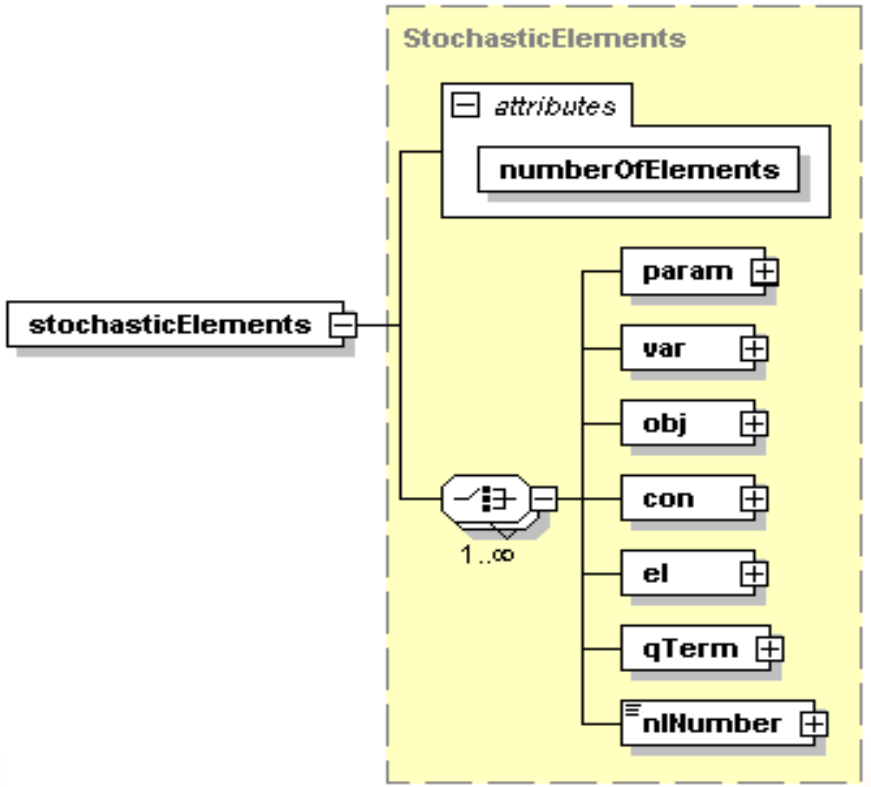
Transformations

- Random variables separated from model entities
- Linked to stochastic problem elements by transformations (linear or nonlinear)
- Useful for factor models and other stochastic processes



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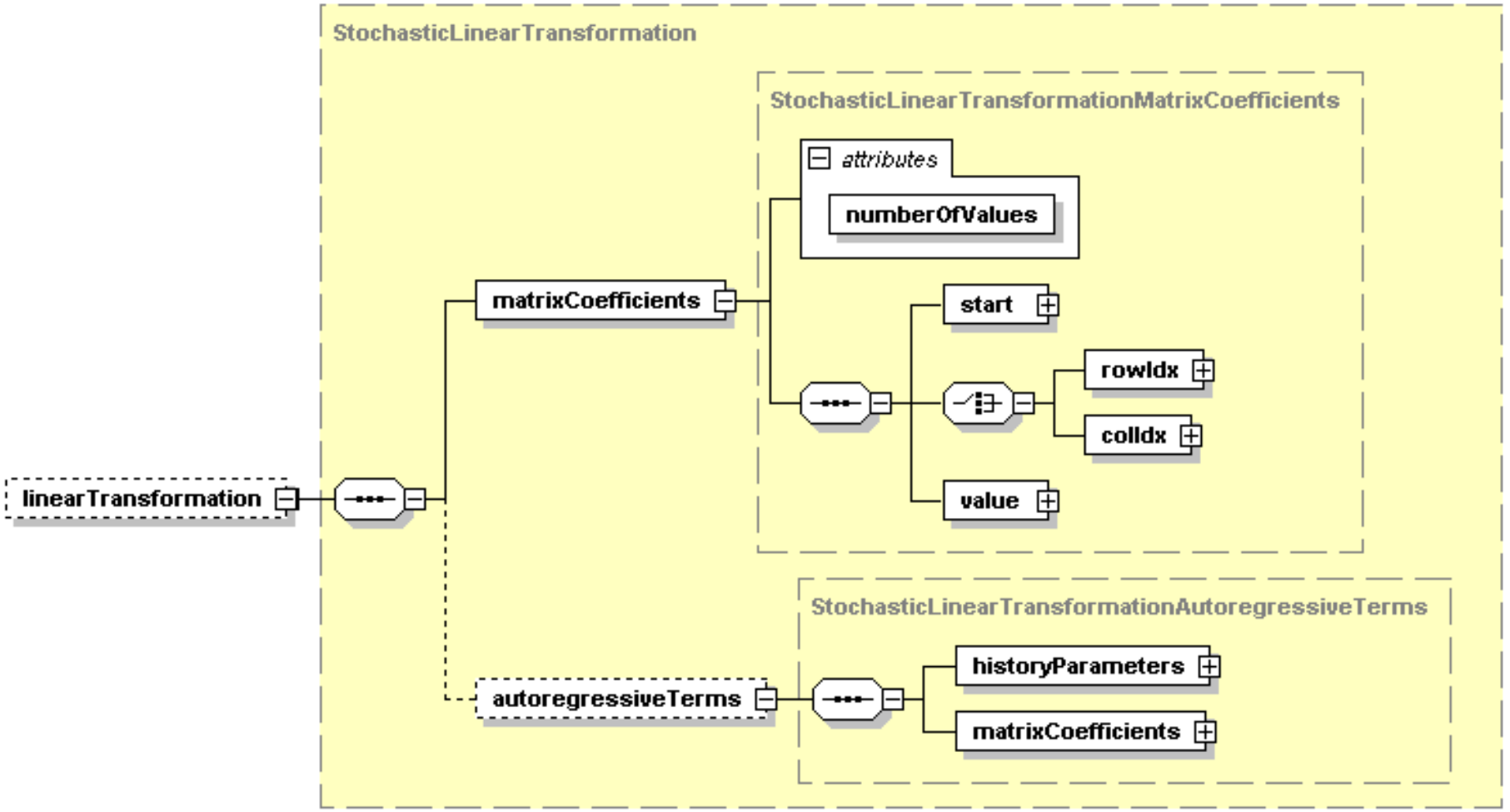
OSiL schema – Linear transformations



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Linear transformation – Example

```
<stochasticTransformation>
  <linearTransformation>
    <stochasticElements
        numberOfElements="6">
      <el rowIdx="1" colIdx="0"/>
      <el rowIdx="1" colIdx="1"/>
      <el rowIdx="2" colIdx="2"/>
      <el rowIdx="2" colIdx="3"/>
      <el rowIdx="3" colIdx="4"/>
      <el rowIdx="3" colIdx="5"/>
    </stochasticElements>
    <matrixCoefficients
        numberOfElements="6">
      <start>
        <el>0</el>
        <el>1</el>
        <el>2</el>
        <el>3</el>
        <el>4</el>
        <el>5</el>
        <el>6</el>
      </start>
      <rowIdx>
        <el>0</el>
        <el>1</el>
        <el>2</el>
        <el>3</el>
        <el>4</el>
        <el>1</el>
      </rowIdx>
      <value>
        <el mult="6">1.0</el>
      </value>
    </matrixCoefficients>
  </linearTransformation>
</stochasticTransformation>
```

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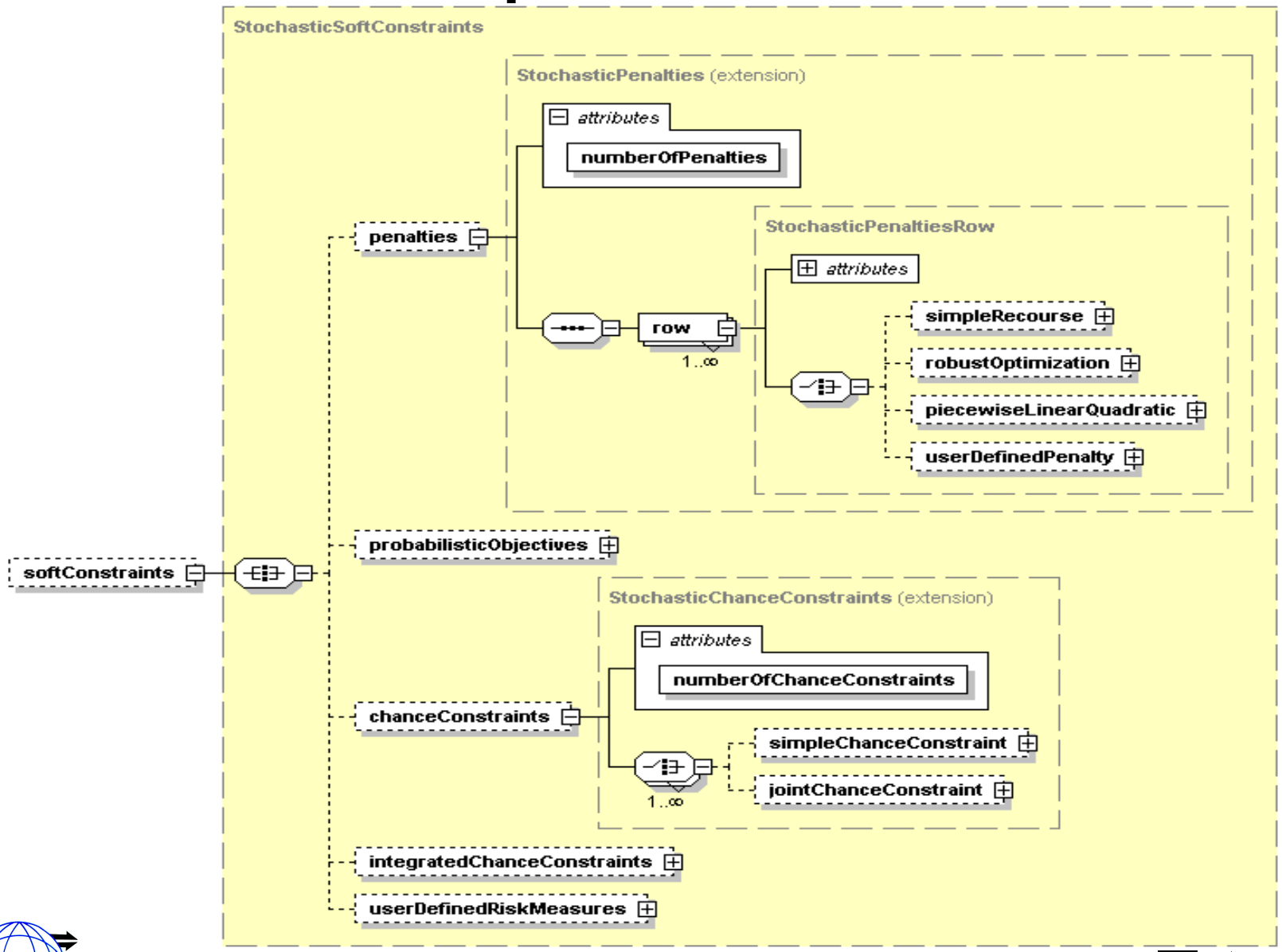
Penalties and probabilistic constraints



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Nonlinear expression – $(2x_0 - x_1^2)^2 + (1 - x_0)^2$

```
<plus>
   <power>
      <minus>
         <var coef="2.0" idx="0"/>
         <power>
            <var coef="1.0" idx="1"/>
            <number value="2"/>
         </power>
      </minus>
      <number value="2"/>
   </power>
   <power>
      <minus>
         <number value="1"/>
         <var coef="1.0" idx="0"/>
      </minus>
      <number value="2"/>
   </power>
</plus>
```

Capabilities

- Arbitrary nonlinear expressions
- Arbitrary distributions
- Scenario trees
- Stochastic problem dimensions
- Simple recourse
- Soft constraints with arbitrary penalties
- Probabilistic constraints
- Arbitrary moment constraints

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Further work

- Internal data structures (OSInstance)
- OSiLWriter: SMPS → OSiL
- Library of problems (netlib, POSTS, Ariyawansa and Felt, Watson, SIPLib,…)
- Readers
- Solver interfaces
- Buy-in

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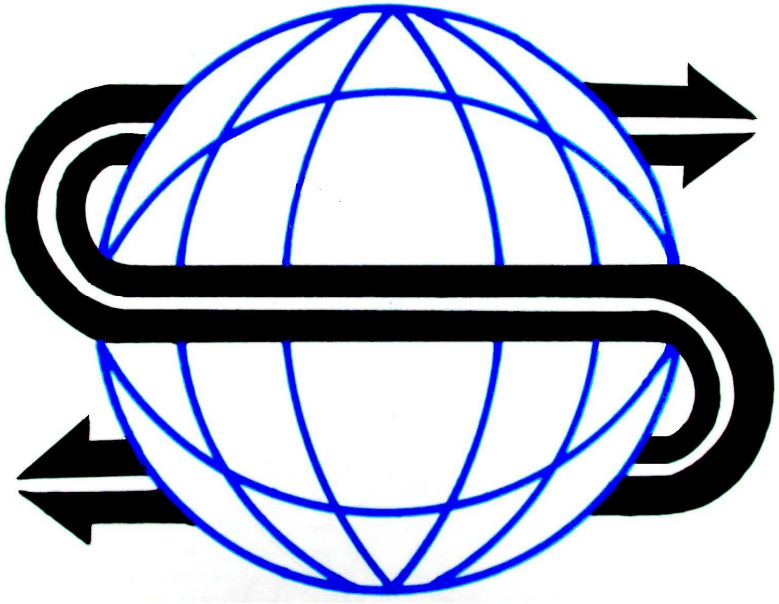
QUESTIONS?



http://www.optimizationservices.org

http://myweb.dal.ca/gassmann

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