Data requirements for stochastic solvers

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Overview

- Introduction
- Algorithms
- Data structures
- Conclusions
Multistage stochastic linear program

\[
\begin{align*}
\min & \quad c_0 x_0 + c_1 x_1 + K + c_T x_T \\
\text{s.t.} & \quad A_{00} x_0 \quad \sim b_0 \\
& \quad R_1 x_0 \quad \Delta r_1 \\
& \quad A_{10} x_0 + A_{11} x_1 \quad \Delta b_1 \\
& \quad M M O M \\
& \quad A_{T0} x_0 + A_{T1} x_1 + K + A_{TT} x_T \quad \Delta b_T \\
& \quad l_0 \leq x_0 \leq u_0 \\
& \quad l_t \leq x_t \leq u_t, \quad t = 1, K, T
\end{align*}
\]

Any data item with nonzero subscript may be random (including dimensions where mathematically sensible)

\(\sim\) stands for arbitrary relation \((\leq, =, \geq)\)
Constraints involving random elements

\[ A_{t0} x_0 + A_{t1} x_1 + K + A_{tt} x_t \Delta b_t \]

\( \Delta \) means

- with probability 1
- or with probability at least \( \beta \)
- or with expected violation at most \( \nu \)
Problem classes

- Recourse problems
  - All constraints hold with probability 1

- Chance-constrained problems
  - Typically single stage

- Hybrid problems
  - Recourse problems including probabilistic constraints (VaR) or integrated chance constraints (CVaR)
  - Regulatory necessity
  - Often modelled using integer variables and/or linking constraints
Algorithms

- Direct solution of the deterministic equivalent
  - “Curse of dimensionality”

- Decomposition
  - Recognize structure
  - Repeated calls to solver with different data
  - Sampling of scenarios during algorithm
    - stochastic decomposition
    - successive approximation
    - “EVPI relaxation”
  - Scenario generator between AML and solver
Data Structures

- Often $O(10^6)$ variables and constraints
- Most compact representation possible
  - Packed matrix format is insufficient
  - Blocks corresponding to nodes in the event tree
  - Change blocks if problem dimensions are deterministic
  - $A_{stj} = A_{st0} + \Delta A_{stj}$ (addition or replacement)
Conclusions

- Stochastic extensions are difficult
- Time is right