Extensions to the OSiL schema: Matrix and cone programming

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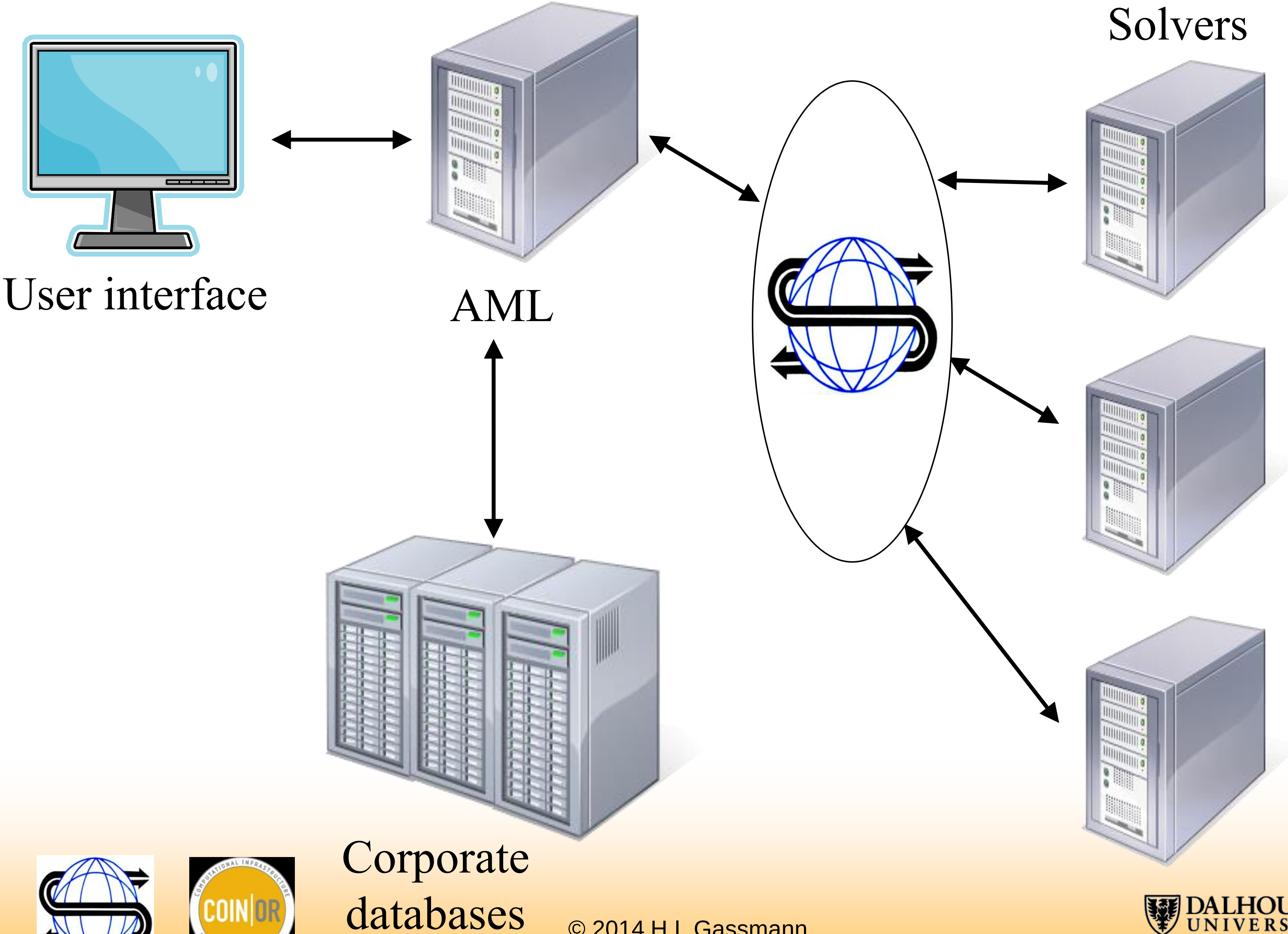
Outline

- Optimization Services
- OSiL (Optimization Services instance Language)
- Matrix and cone programming
- Extensions to OSiL
  - Matrices
  - Cones
  - Matrix programming
- OS and CSDP

Optimization Services

- Framework for optimization in distributed computing environment
- XML schemas for communicating instances, options, results, …
- Implementation (COIN-OR project OS)
  - OSSolverService
  - OSAmplClient
  - OSServer

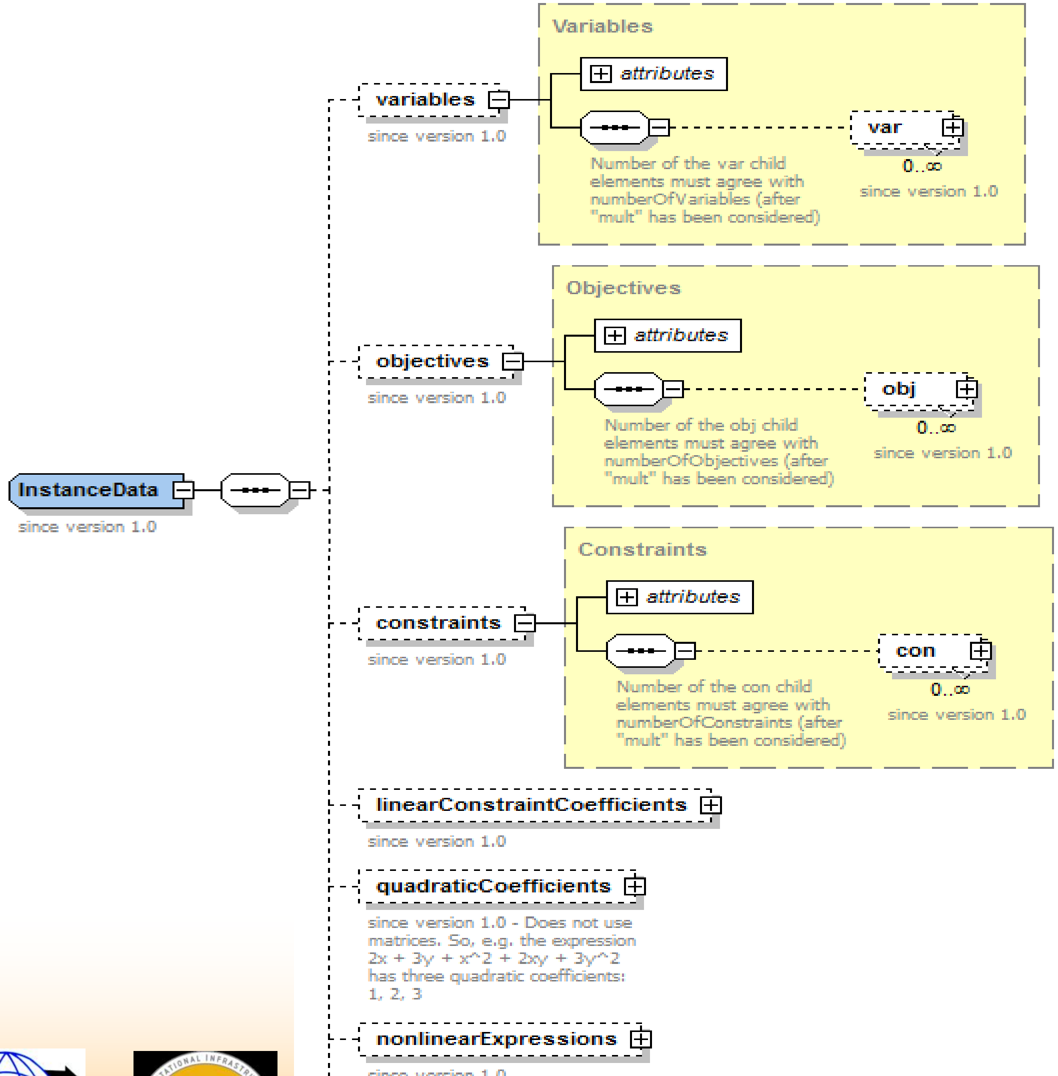

User interface
AML
Solvers
Corporate
databases

Core OS and OSiL

- Problems handled
  - Linear programs
  - Integer programs
  - MILP
  - Convex NLP
  - Discrete NLP
  - Nonconvex NLP

- Available solvers
  - Clp
  - Cbc
  - SYMPHONY
  - Ipopt
  - Bonmin
  - Couenne
  - Glpk
  - Cplex
  - Gurobi
  - Mosek

“Core” OSiL elements



Variable types (C,I,B)
Upper and lower bounds

Possibly multi-objective
Maximize or minimize
Linear objective coefficients

Upper and lower bounds

Matrix and cone programming

- Constraints (and objectives) expressed in terms of cones
  - Second order cones
  - Cones of positive semidefinite matrices
  - Orthant cones
  - Cones of nonnegative polynomials (over some interval)
- Solvers: CSDP, SeDuMi, Mosek, Cplex, Gurobi

Sample problems

- Second order cone program

$$\min \ f'x$$
$$\text{s.t.} \ \|A_i x + b_i\|_2 \le c_i' x + d_i$$
$$Fx = g$$

- Semidefinite program

$$\min C \cdot X$$
$$\text{s.t.} \ \mathcal{A}X = b$$
$$X \ \text{symmetric, positive semidefinite}$$

Requirements and challenges

- Cone inclusions, e.g.,
  - X is symmetric, positive semidefinite
  - Ax lies in the rotated quadratic cone RC(1,2;3..n)
- Matrix expressions, e.g.,
  - $AXB + BXA$
  - trace A^TX
- What does a linear matrix expression look like?

Design principles

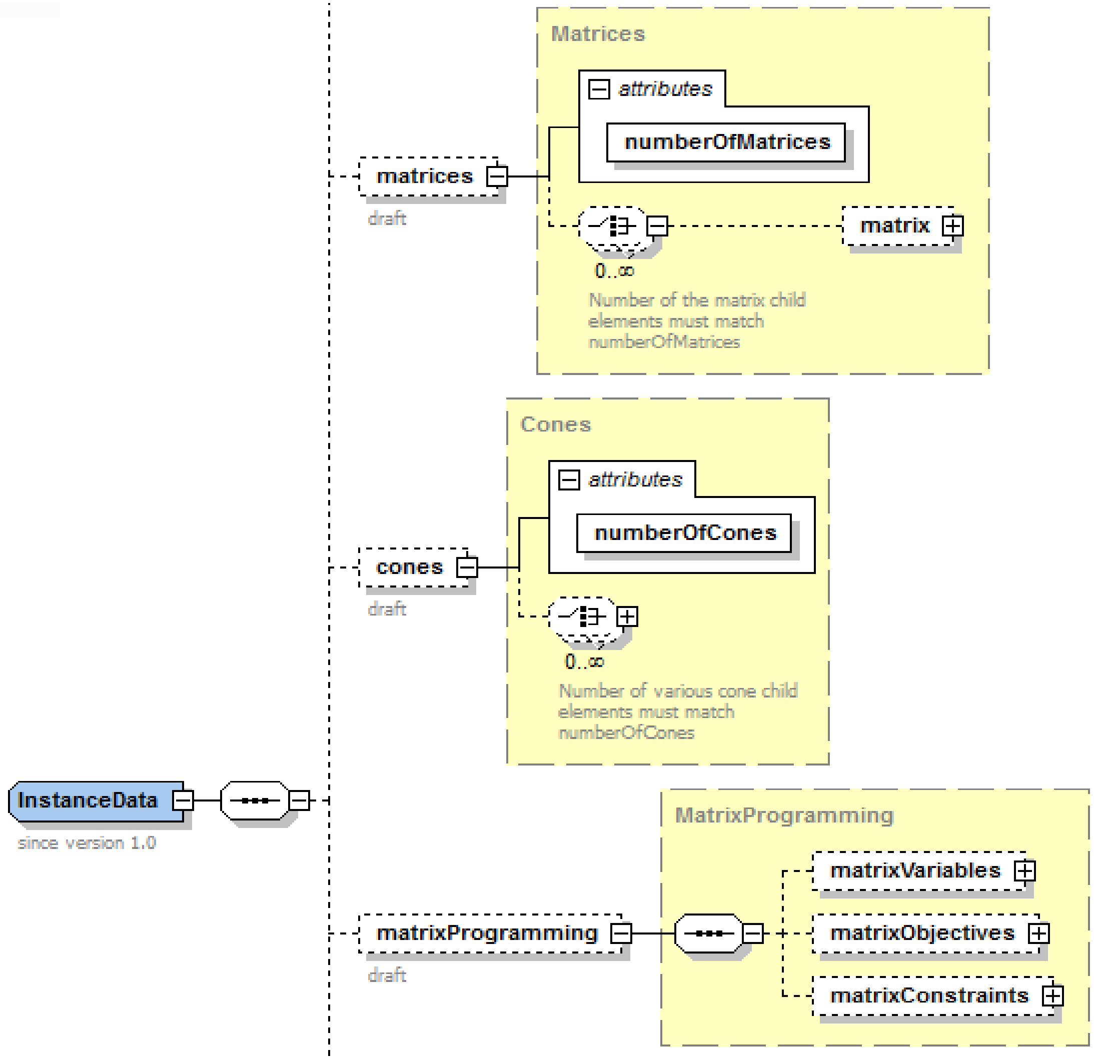
- Preserve core
- Be as general as possible
- Respect sparsity
- Avoid painting yourself into corners
- New constructs
  - <Matrix>
  - <Cone>
  - <MatrixProgramming>

OSiL and matrices

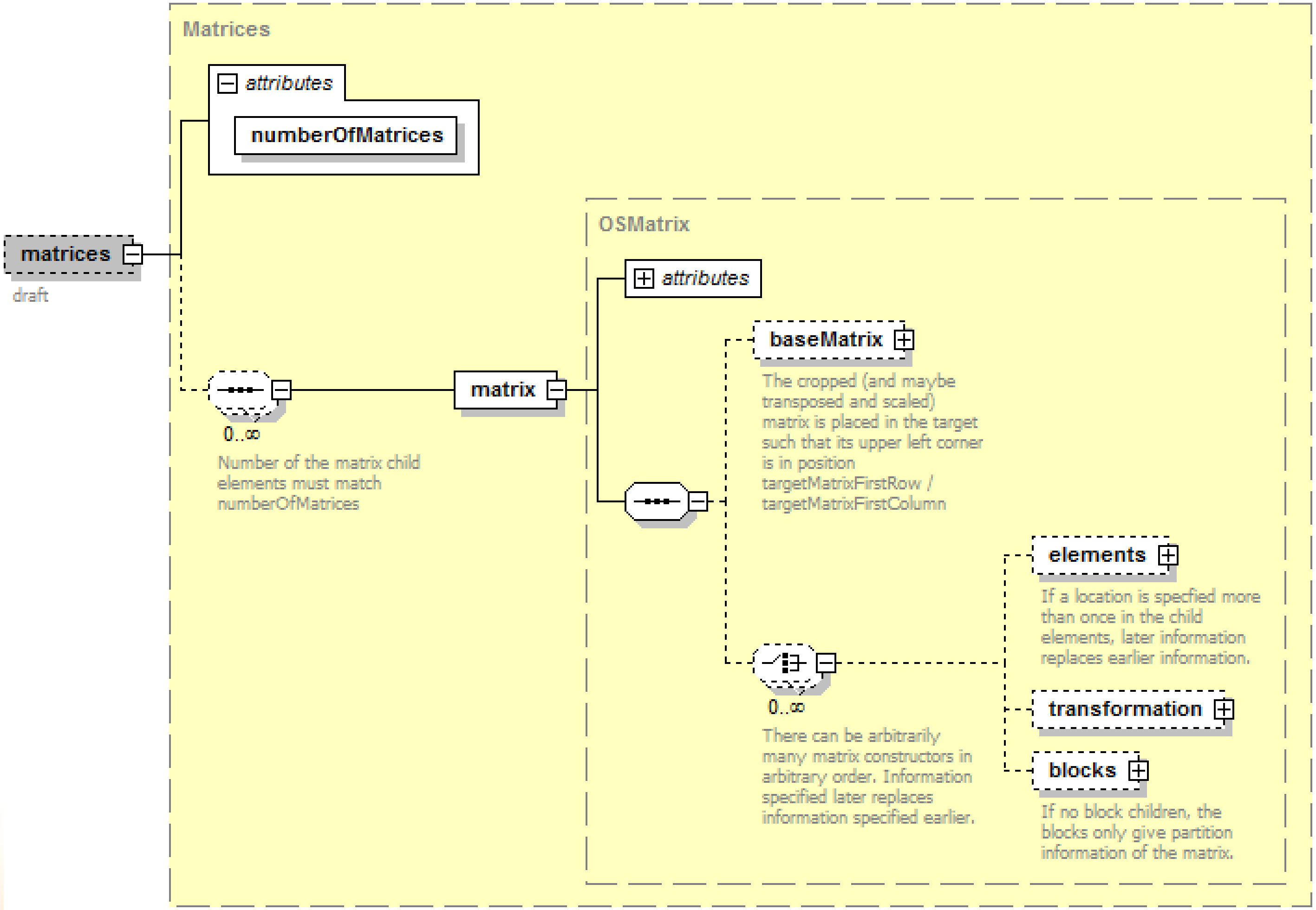
- Constant matrices $\begin{pmatrix} 1 & 3 & 6 \\ -1 & 0 & 4 \end{pmatrix}$
- Matrix variables $\mathcal{X}$ (or $\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}$?)
- General matrices (e.g., Hessian, Jacobian, etc.) $\begin{pmatrix} x^2 & 5 & 0 & 1-\ln(\sin x) \end{pmatrix}$

- Symmetry
- Sparsity
- Block structure
- Matrix construction, e.g., $A = a\, a^T$
- One matrix type or several?

OSiL: Matrix and cone extensions



The <matrices> element



Example 1 – elements

```
<matrices numberOfMatrices="4">
    <matrix numberOfRows="2" numberOfColumns="1">
        <elements>
            <constantElements>
                <start numberOfEl="2">
                    <el> 0 </el>     <el> 2 </el>
                </start>
                <nonzeros numberOfEl="2">
                    <indexes>
                        <el> 0 </el>     <el> 1 </el>
                    </indexes>
                    <values>
                        <el> 1 </el>     <el> 2 </el>
                    </values>
                </nonzeros>
            </constantElements>
        </elements >
    </matrix>
```

$$M_0 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Example 2 - transformation

```
<matrix numberOfRows="2" numberOfColumns="2">
    <transformation>
        <matrixTimes>
            <matrix matrixIdx="0"/>
            <matrixTranspose>
                <matrix matrixIdx="0"/>
            </matrixTranspose>
        </matrixTimes>
    </transformation>
</matrix>
```

$$M_1 = M_0 M_0^T$$

$$= \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

Example 3 - blocks

```
<matrix numberOfRows="3" numberOfColumns="4">
    <blocks numberOfBlocks="2">
        <colOffsets numberOfEl="2">
            <el> 0 </el>    <el> 2</el>
        </colOffsets>
        <rowOffsets numberOfEl="2">
            <el> 0 </el>    <el> 2</el>
        </rowOffsets>
        <block blockRowIdx="0" blockColIdx="0">
            <baseMatrix baseMatrixIdx="1"/>
        </block>
        <block blockRowIdx="1" blockColIdx="1">
            <baseMatrix baseMatrixIdx="0" baseTranspose="true" scalarMultiplier="-3"/>
        </block>
    </blocks>
</matrix>
```

$$M_2 = \begin{pmatrix} M_1 & 0 \\ 0 & -3M_0^T \end{pmatrix} = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ \hline 0 & 0 & -3 & -6 \end{array}\right)$$

Example 4 – base matrix

```
<matrix numberOfRows="5" numberOfColumns="5">
   <baseMatrix
            baseMatrixIdx="2"
            baseMatrixFirstRow="0"
            baseMatrixFirstCol="0"
            baseMatrixLastRow="2"
            baseMatrixLastCol="2"
            baseTranspose="false"
            scalarMultiplier="1.0"
            targetMatrixFirstRow="1"
            targetMatrixLastRow ="1"
         />
</matrix>
```

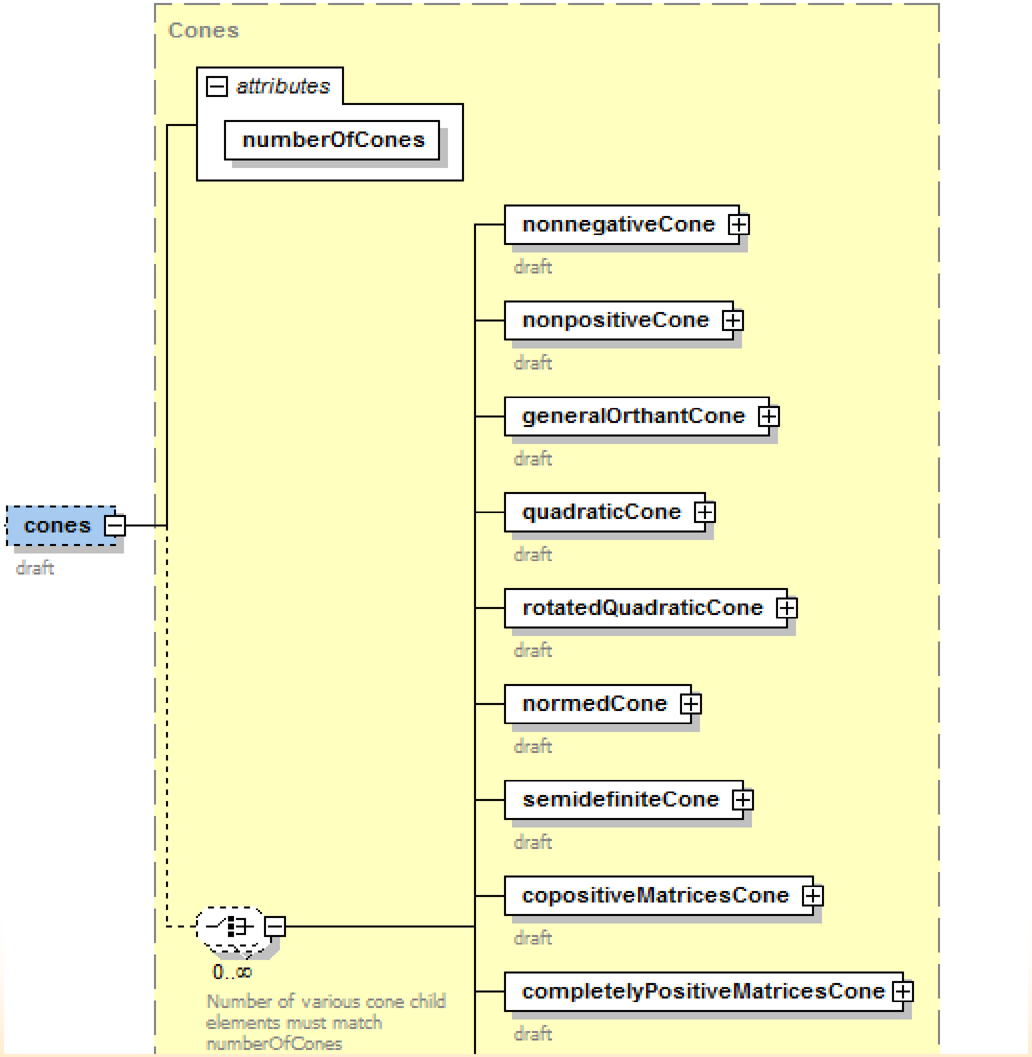
$$M_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Example 5 – variable references

```
<matrix numberOfRows="2" numberOfColumns="2" symmetry="lower">
  <elements>
    <varReferenceElements>
      <start numberOfEl="3">
        <el> 0 </el>    <el> 2 </el>    <el> 3 </el>
      </start>
      <nonzeros numberOfEl="3">
        <indexes>
          <el> 0 </el>    <el> 1 </el>    <el> 1 </el>
        </indexes>
        <values>
          <el> 1 </el>    <el> 2 </el>    <el> 3 </el>
        </values>
      </nonzeros>
    </varReferenceElements >
  </elements >
</matrix>
```

$$M_4 = \begin{pmatrix} x_1 & x_2 \\ x_2 & x_3 \end{pmatrix}$$

The <cones> element

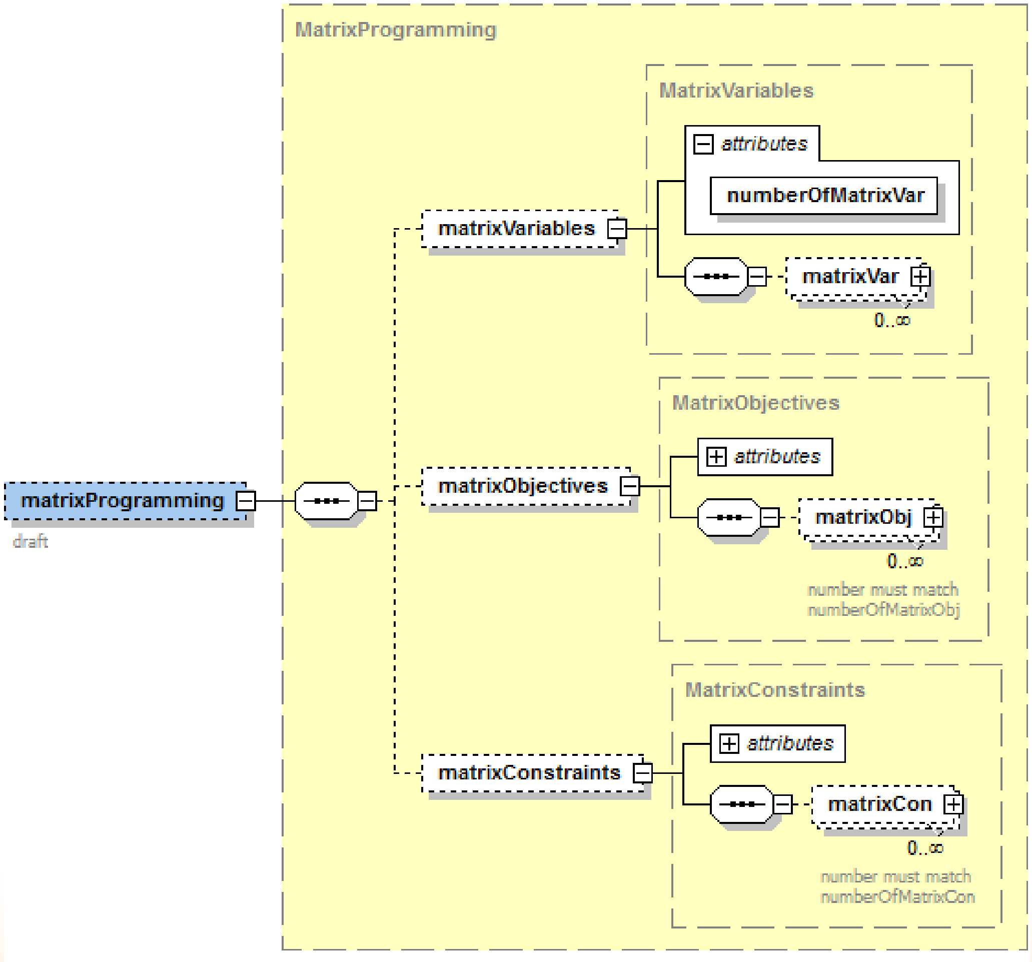


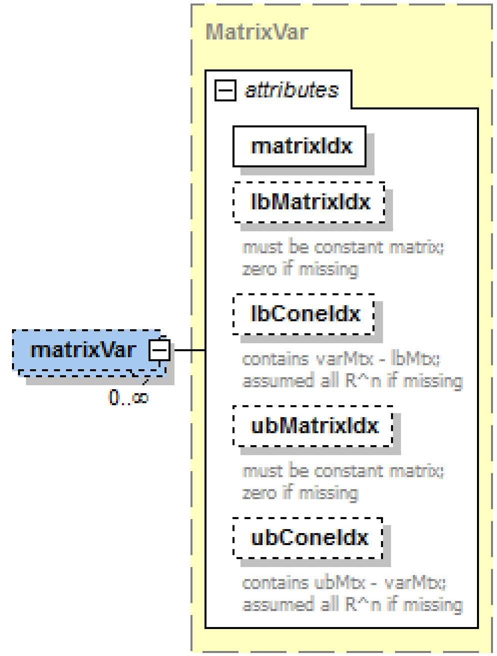
…

The <cones> element - Example

```
<cones numberOfCones=“4">
   <semidefiniteCone numberOfRows= "2" numberOfCols= "2"/>
   <quadraticCone numberOfRows= "4" numberOfCols= "1"/>
   <nonnegativeCone numberOfRows= "3" numberOfCols= "1"/>
   <nonnegativePolynomialCone maxDegree="4"
         numberOfRows= "2" numberOfCols= "1"/>
</cones>
```

<matrixProgramming>





M_4 is positive semidefinite (i.e., in C_0):

<matrixVar matrixIdx="4" lbConeIdx="0"/>

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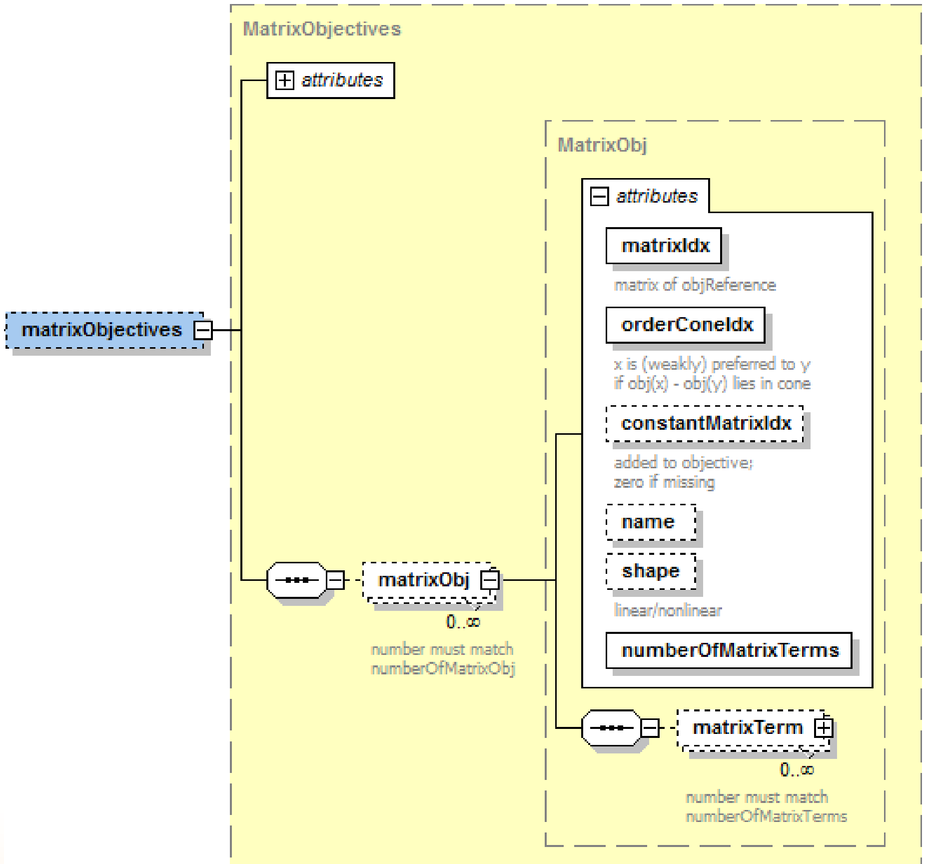
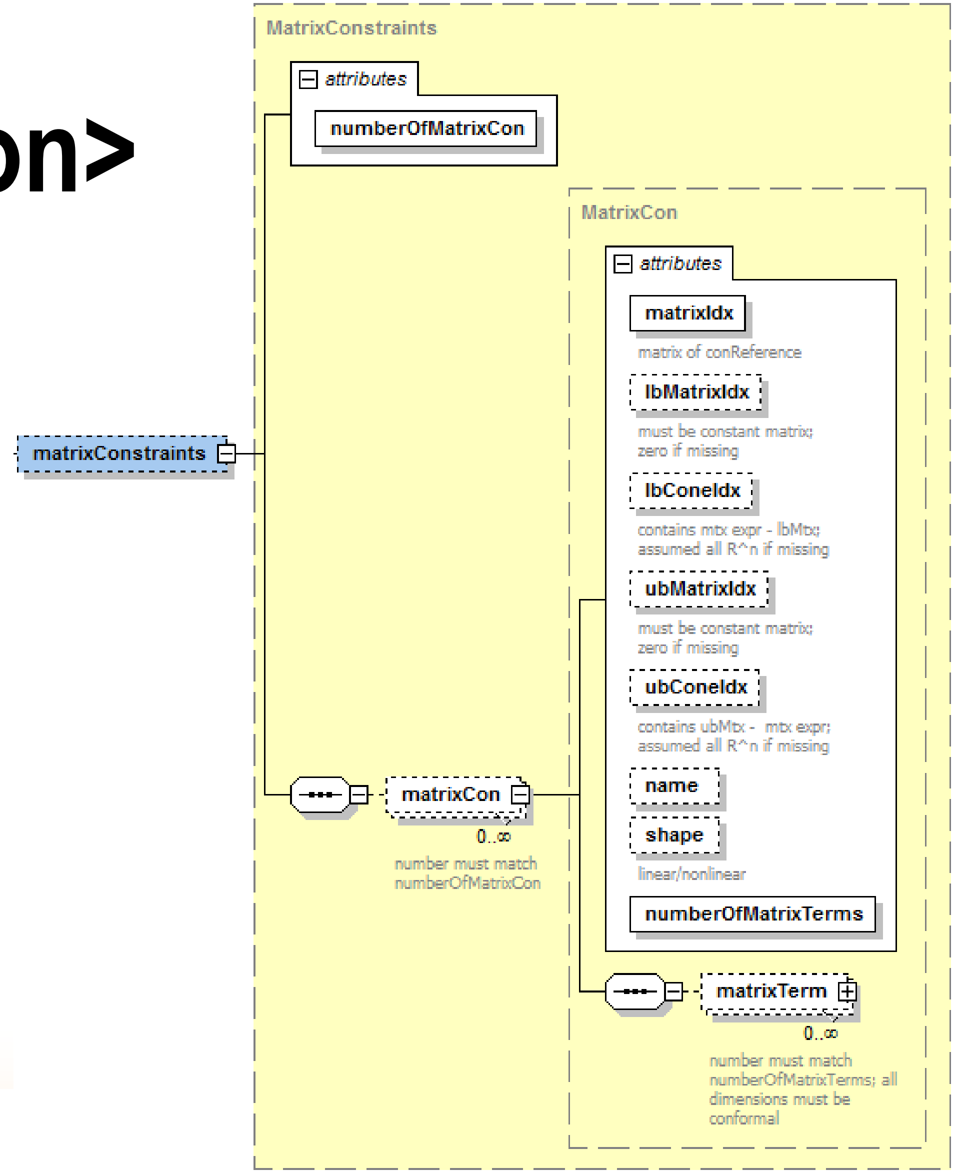
<matrixObj> and <matrixCon>





Future work

- Complete the parser
- Translate SDP problems (SDPlib)
- Write CSDP driver

CSDP

- Open-source project (COIN-OR)
- Solves

$$\begin{aligned} \max \quad & \mathrm{tr}(CX) \\ \mathrm{tr}(A_1 X) &= a_1 \\ \mathrm{tr}(A_2 X) &= a_2 \\ & \cdots \\ \mathrm{tr}(A_m X) &= a_m \\ X &\succeq 0 \end{aligned}$$

- Assumes A_i, C, X are real and symmetric
- $\succeq$: positive semidefinite

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How to get OS

- Download
  - Binaries
    - http://www.coin-or.org/download/binary/OS
      - OS-2.1.1-win32-msvc9.zip
      - OS-2.3.0-linux-x86_64-gcc4.3.2.tgz
  - Stable source
    - http://www.coin-or.org/download/source/OS/
      - OS-2.8.0.tgz
      - OS-2.8.0.zip
  - Development version (using svn)
    - svn co https://projects.coin-or.org/svn/OS/releases/2.8.0 COIN-OS
    - svn co https://projects.coin-or.org/svn/OS/trunk COIN-OS

QUESTIONS?

http://www.optimizationservices.org

https://projects.coin-or.org/OS

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