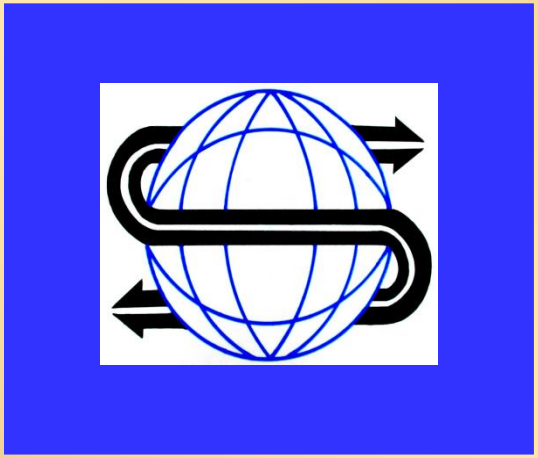

Recent developments in Optimization Services (OS)

Horand I. Gassmann, Dalhousie University

Jun Ma, Kipp Martin, Imre Pólik

Outline

- Optimization Services
- Matrices and cones
  - Schema representation
  - Semidefinite programming
  - CSDP
- Modifications, real-time and stochastic data
- Conclusions

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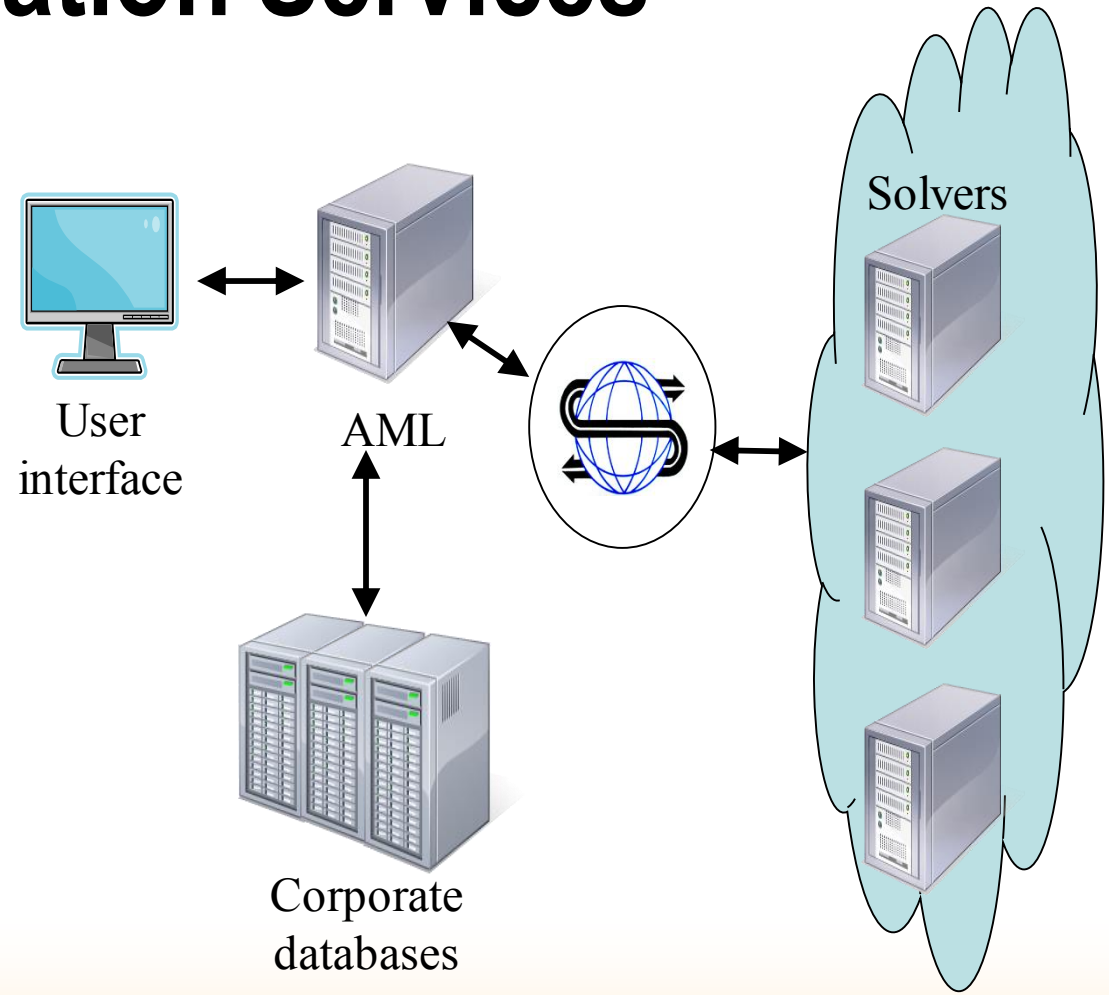
Optimization Services

- Framework for optimization in a distributed computing environment or in a compute cloud
- XML schemas for communicating
  - instances (OSiL)
  - options (OSoL)
  - results (OSrL)
- Implementation (COIN-OR)
- Connects to COIN-OR and other third-party linear, integer and nonlinear solvers
  - Clp, Cbc, Ipopt, SYMPHONY, …
  - Glpk, Cplex, Gurobi, Matlab, …
  - CSDP



“Core” OS and OSiL

- Problems handled
  - Linear programs
  - Integer programs
  - MILP
  - Convex NLP
  - Discrete NLP
  - Nonconvex NLP
- Available solvers
  - Clp
  - Cbc
  - SYMPHONY
  - Ipopt
  - Bonmin
  - Couenne
  - Glpk
  - Cplex
  - Gurobi
  - Mosek

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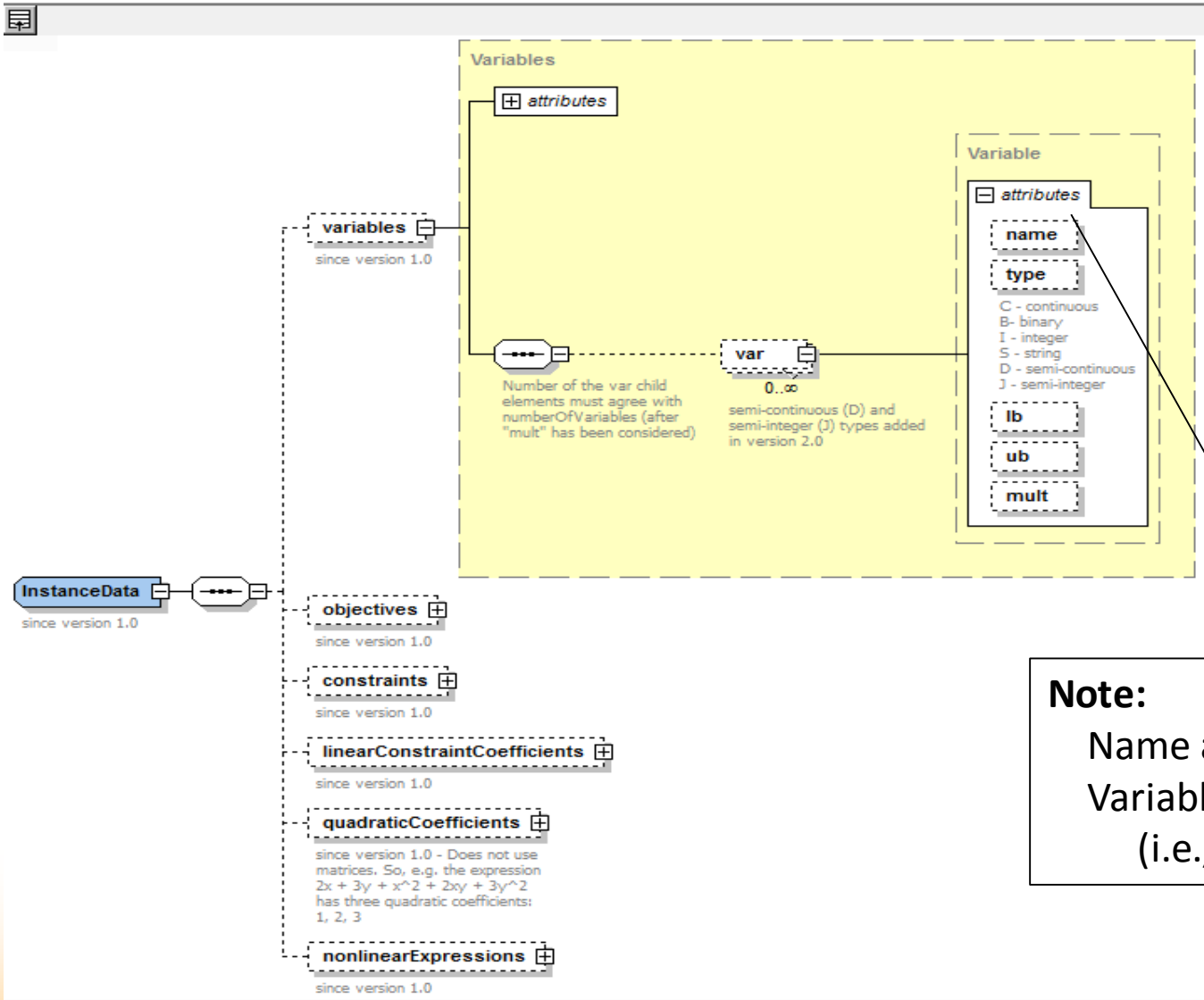
OSiL - “Core” elements




```
<variables
  numberOfVariables
    ="10">
  <var mult="10"
    ub="1000"/>
</variables>
```

Sets up 10 continuous variables with lb=0 and ub=1000

Note:

Name attribute is optional

Variables identified by index (i.e., position number)

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Matrices in optimization

- Coefficient matrices in linear programming
- Jacobian and Hessian matrices, gradient vectors
- Matrix variables in positive semidefinite programming

- Constant matrices $\begin{pmatrix} 1 & 3 & 6 \\ -1 & 0 & 4 \end{pmatrix}$
- Matrix variables $\mathcal{X}$ (or $\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}$?)
- General matrices (e.g., Hessian, Jacobian, etc.) $\begin{pmatrix} x^2 & 5 & 0 & 1-\ln(\sin x) \end{pmatrix}$
- Vectors are matrices, too…

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Matrix representations

- Formats for sparse matrix representations
  - Sparse column format (starts, indices and values)
  - MPS, CPLEX LP format
  - SDPA format

  Static (real) values only

- Sparsity
- Symmetry
- Block structure
- Matrix construction, e.g., $A = a\ a^T$

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Cone programming

- Constraints (and objectives) expressed in terms of cones
  - Second order cones
  - Cones of positive semidefinite matrices
  - Orthant cones (linear cones)
  - Cones of nonnegative polynomials (over some interval)
- Solvers: CSDP, SeDuMi, Mosek, Cplex, Gurobi, …
- Unified treatment within the Optimization Services framework

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Design principles

- Preserve core
- Be as general as possible
- Respect sparsity
- Avoid painting yourself into corners
- New constructs
  - <matrix>
  - <cone>
  - <matrixProgramming>

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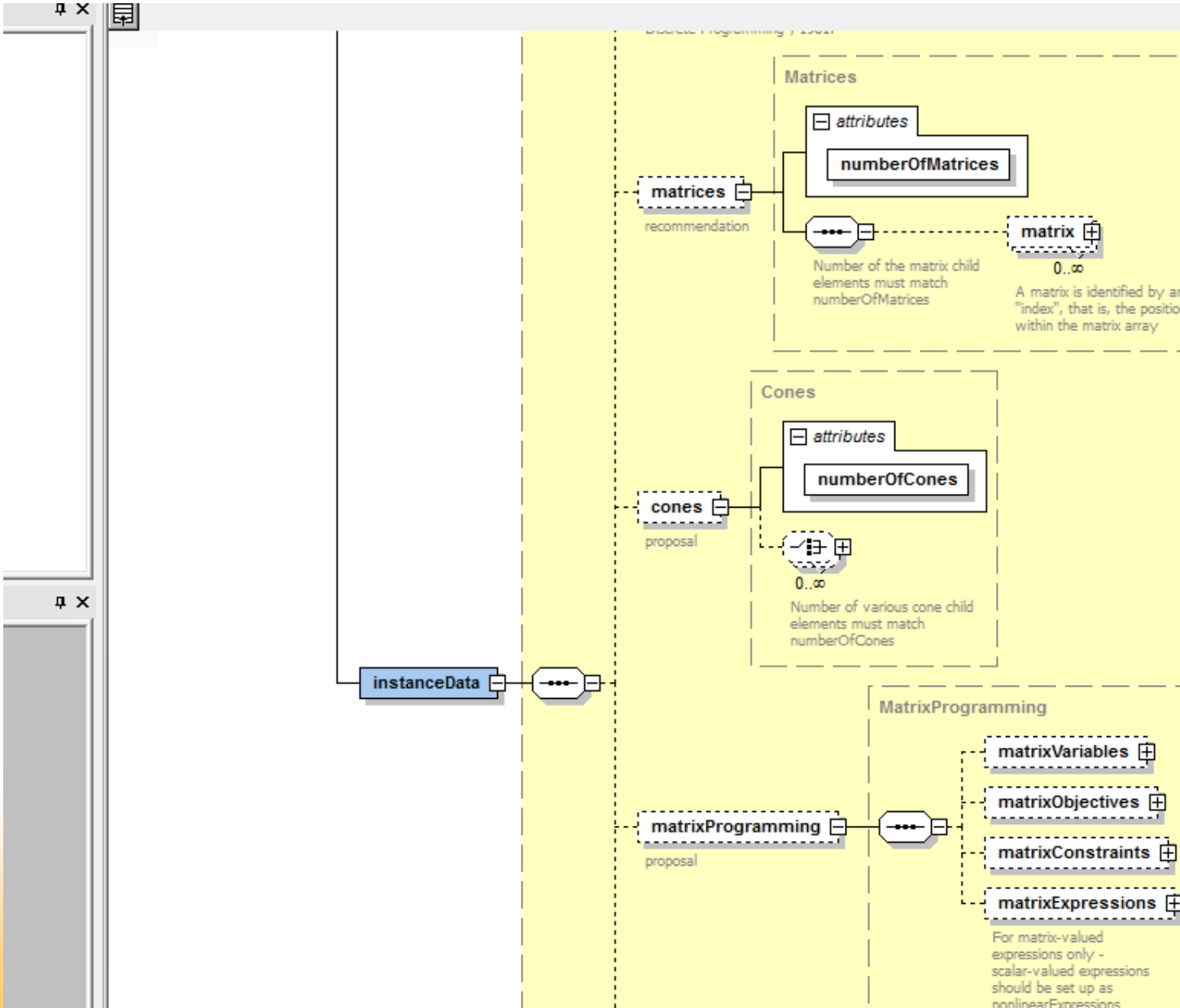
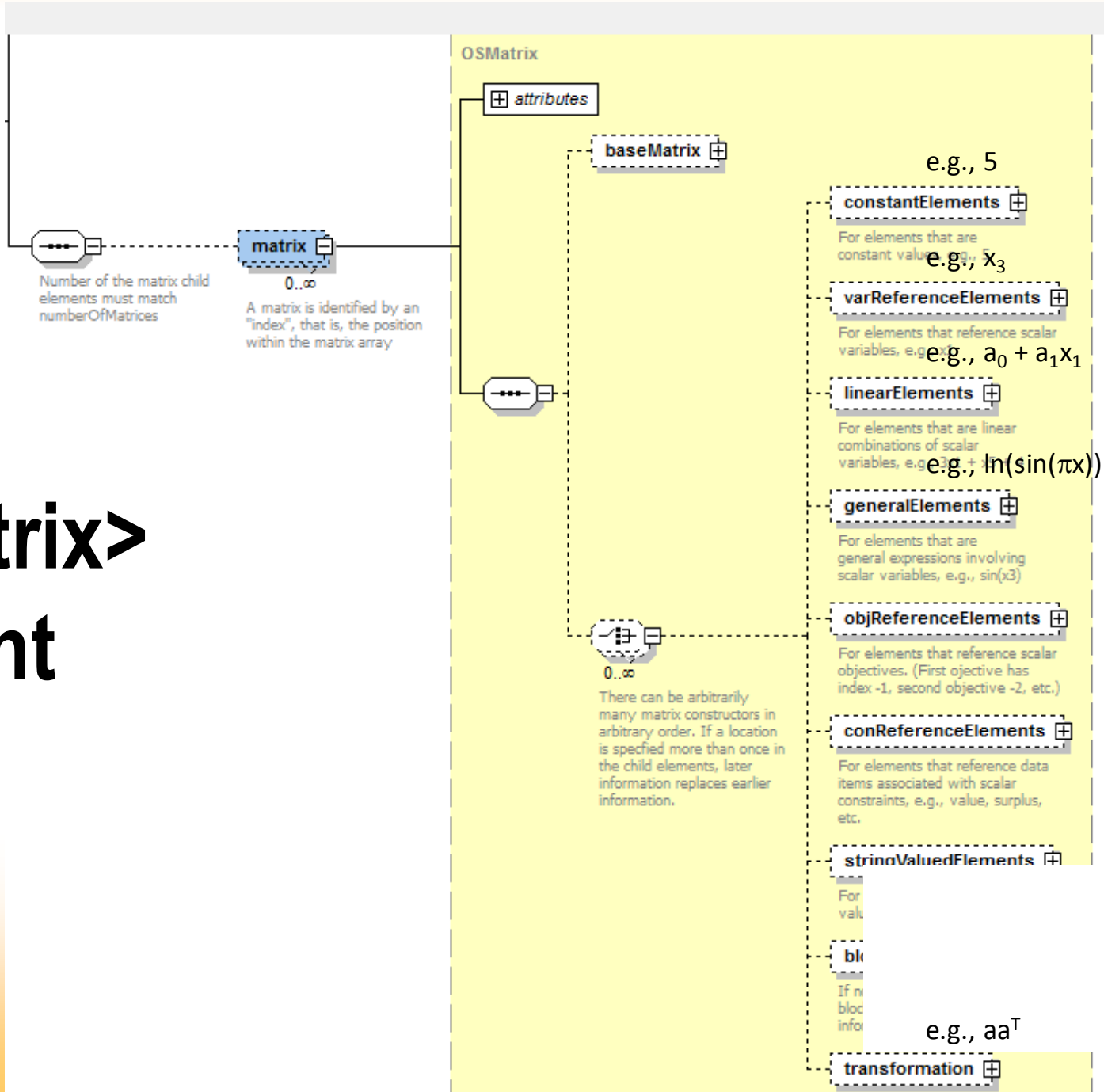
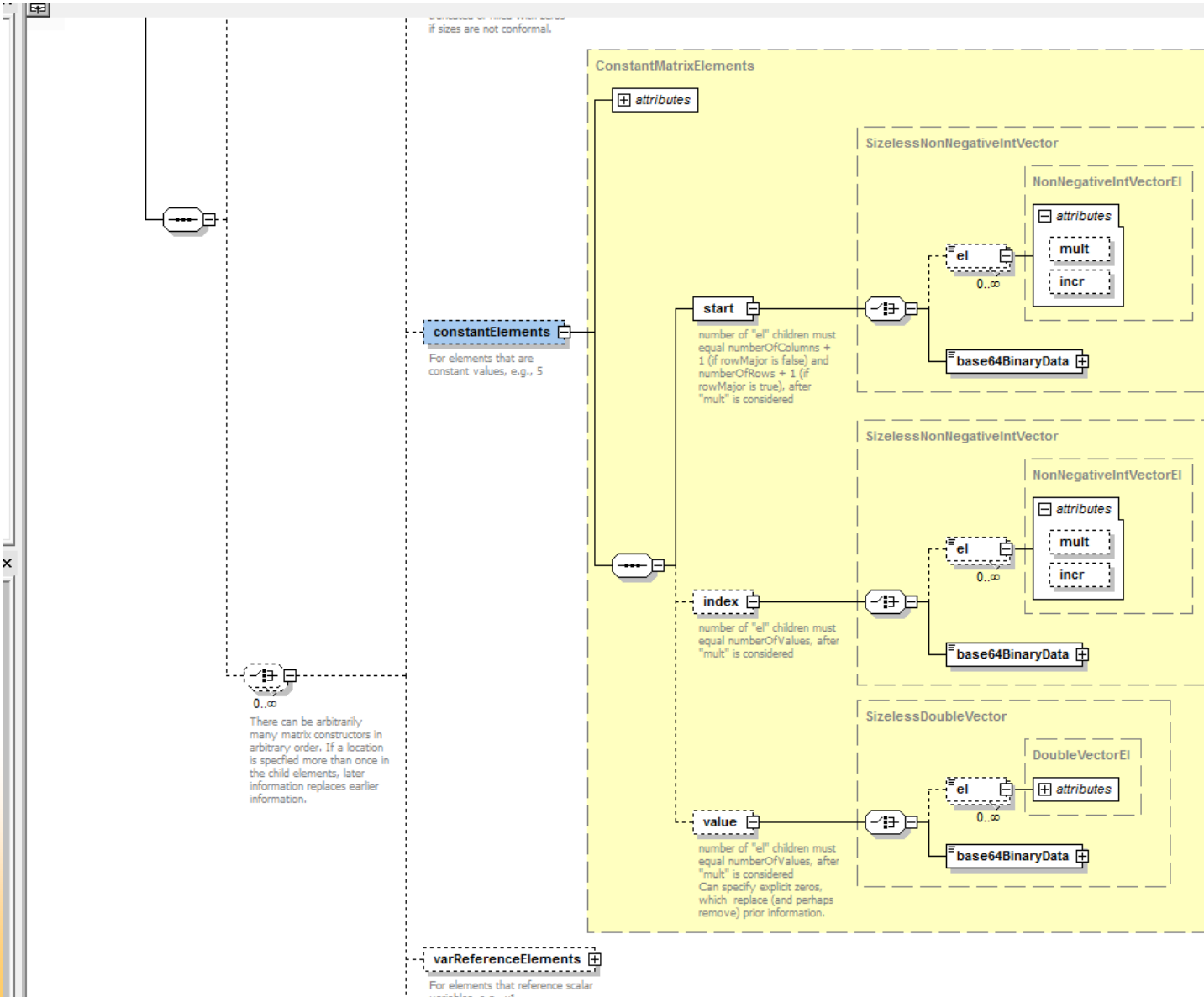
OSiL: Matrix and cone extensions



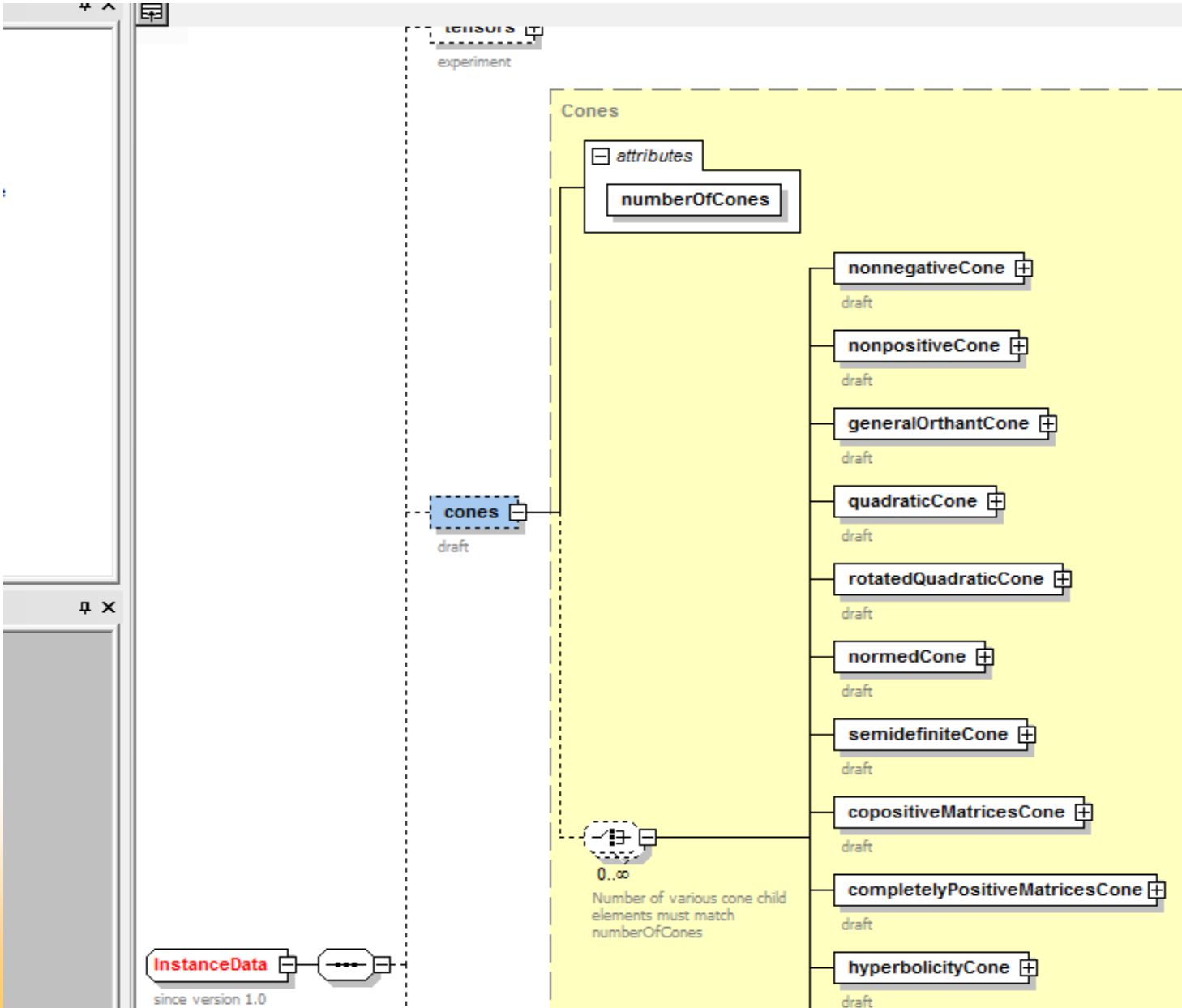
The <matrix> element



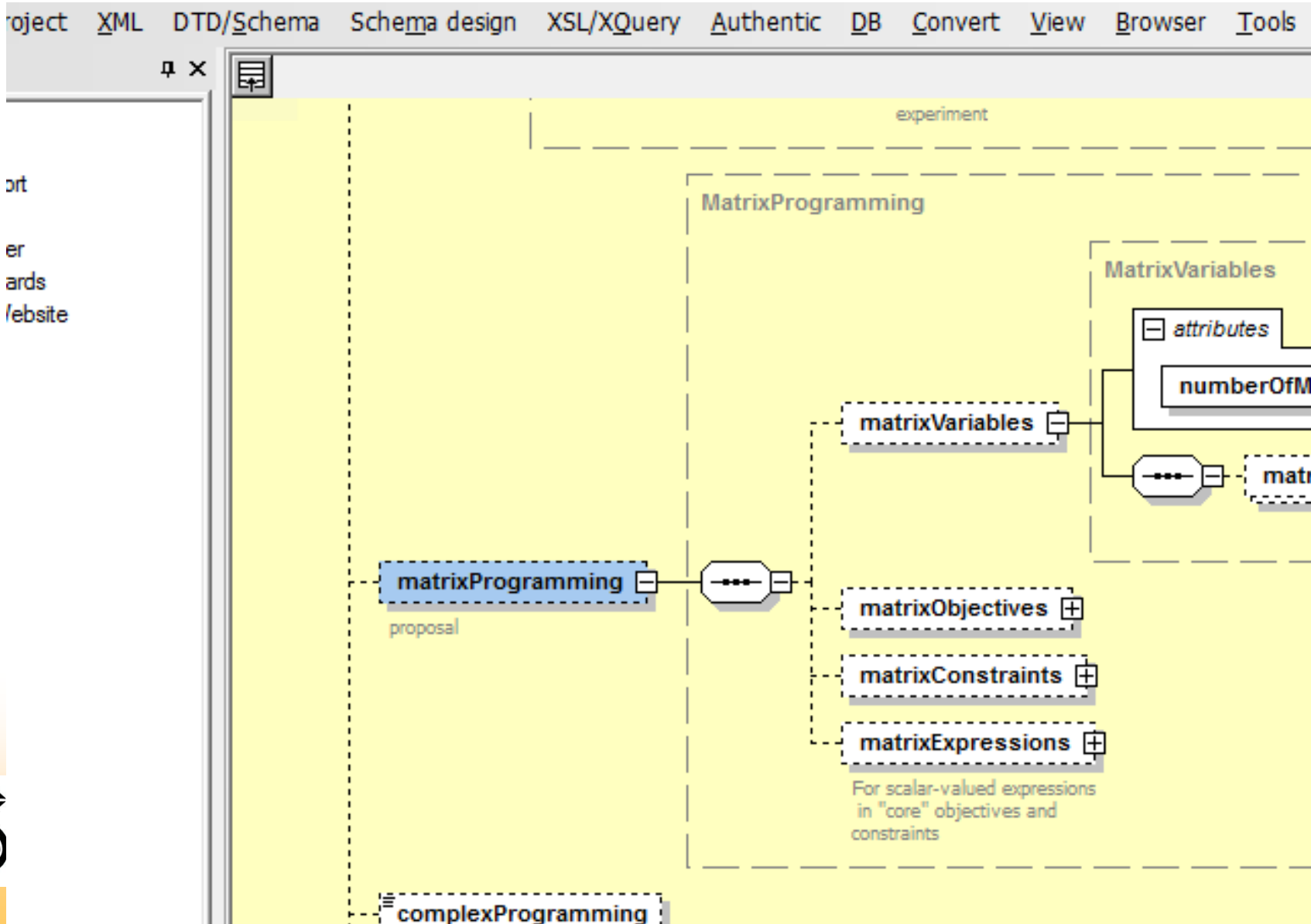
Constant elements (sparsity)

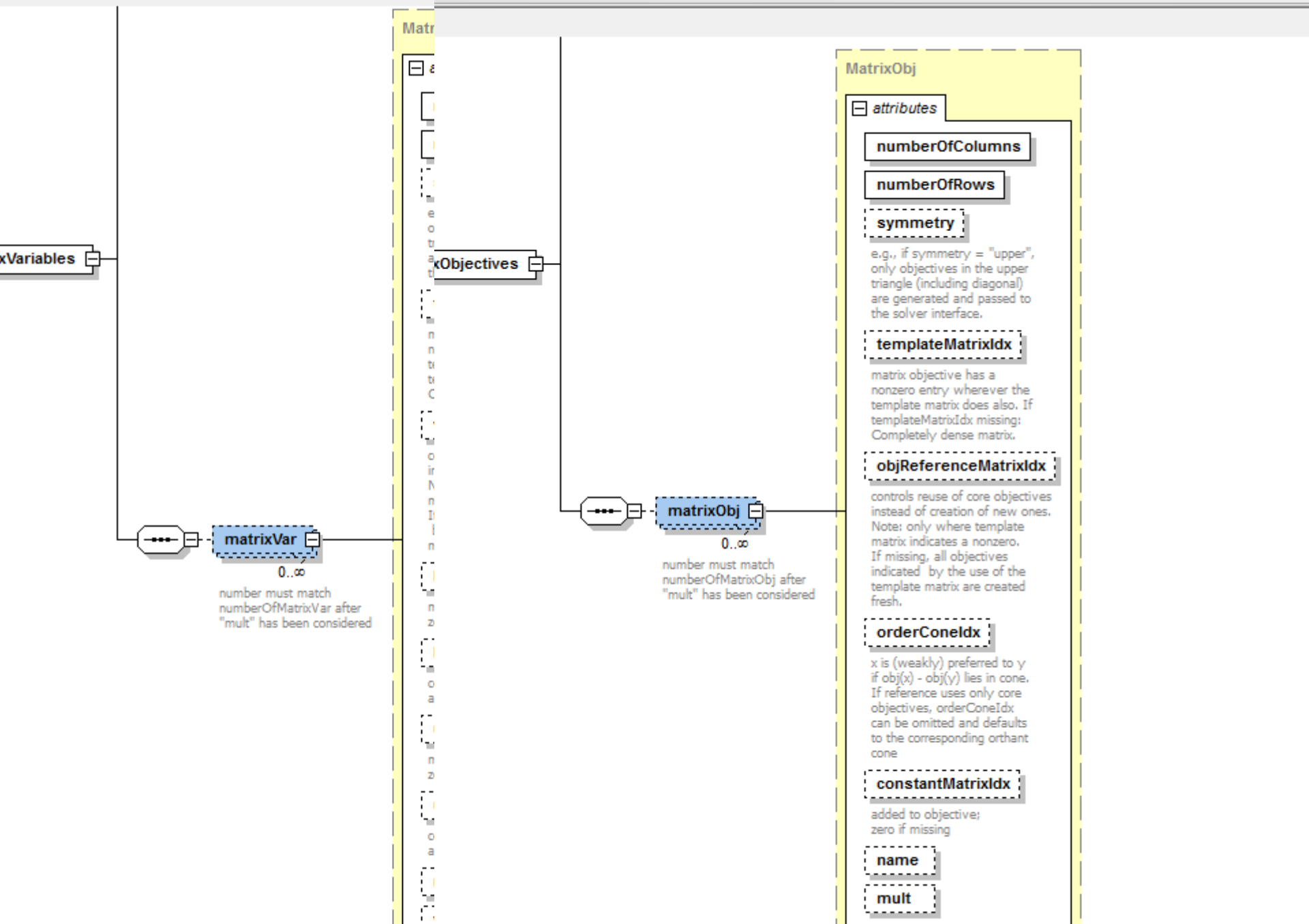


The <cones> element



<matrixProgramming>




xVariables
matrixVar
0..∞
number must match numberOfMatrixVar after "mult" has been considered
xObjectives
matrixObj
0..∞
number must match numberOfMatrixObj after "mult" has been considered
MatrixObj
attributes
numberOfColumns
numberOfRows
symmetry
e.g., if symmetry = "upper", only objectives in the upper triangle (including diagonal) are generated and passed to the solver interface.
templateMatrixIdx
matrix objective has a nonzero entry wherever the template matrix does also. If templateMatrixIdx missing: Completely dense matrix.
objReferenceMatrixIdx
controls reuse of core objectives instead of creation of new ones. Note: only where template matrix indicates a nonzero. If missing, all objectives indicated by the use of the template matrix are created fresh.
orderConeIdx
x is (weakly) preferred to y if obj(x) - obj(y) lies in cone. If reference uses only core objectives, orderConeIdx can be omitted and defaults to the corresponding orthant cone
constantMatrixIdx
added to objective; zero if missing
name
mult

CSDP and SDPA

- Open-source project (COIN-OR)
- Solves $\max \quad \mathrm{tr}(CX)$

$$\begin{aligned} \mathrm{tr}(A_1 X) &= a_1 \\ \mathrm{tr}(A_2 X) &= a_2 \\ &\cdots \\ \mathrm{tr}(A_m X) &= a_m \\ X &\succeq 0 \end{aligned}$$

- $\succeq 0$: positive semidefinite
- SDPA format
- SDPLib

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Comparing SDPA and OSiL (sample.sdpa)

$$
\begin{aligned}
\max \quad & \operatorname{tr}(CX) & & \\
& \operatorname{tr}(A_1 X) & = & \; a_1 \\
& \operatorname{tr}(A_2 X) & = & \; a_2 \\
& & \cdots & \\
& \operatorname{tr}(A_m X) & = & \; a_m \\
& X & \succeq & \; 0
\end{aligned}
$$

Many implicit assumptions:
- Maximization problem
- Matrices have conformal block-diagonal structure
- Data matrices are symmetric
- X is symmetric, positive semidefinite
- Matrix expressions

```
2 =mdim
3 =nblocks
2 3 -2
1.0  2.0
0 1 1 1 2.0
0 1 1 2 1.0
0 1 2 2 2.0
0 2 1 1 3.0
0 2 1 3 1.0
0 2 2 2 2.0
0 2 3 3 3.0
1 1 1 1 3.0
1 1 1 2 1.0
1 1 2 2 3.0
1 3 1 1 1.0
2 2 1 1 3.0
2 2 2 2 4.0
2 2 3 3 5.0
2 2 1 3 1.0
2 3 2 2 1.0
```

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Sample.osil (p.1)

```
<?xml version="1.0" encoding="UTF-8"?>
<osil xmlns="os.optimizationservices.org"
     xmlns:os="os.optimizationservices.org"
     xmlns:xs="http://www.w3.org/2001/XMLSchema"
     xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
     xsi:schemaLocation="os.optimizationservices.org
     http://www.optimizationservices.org/schemas/2.0/OSiL.xsd">
   <instanceHeader>
      <name>SDPA problem sample.sdpa</name>
      <description>Translated from SDPA format using SDPA2OSiL
                  (C) H.I. Gassmann 2010-2015
      </description>
   </instanceHeader>
   <instanceData>
      <objectives>
         <obj maxOrMin="max" numberOfObjCoef="0"/>
      </objectives>
      <constraints numberOfConstraints="2">
         <con lb="1." ub="1."/>
         <con lb="2." ub="2."/>
      </constraints>
```


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Sample.osil (p.2)

```
<nonlinearExpressions numberOfNonlinearExpressions="3">
  <nl idx="-1">
    <matrixTrace>
      <matrixTimes>
        <matrixReference idx="0"/>
        <matrixVar idx="0"/>
      </matrixTimes>
    </matrixTrace>
  </nl>
  <nl idx="0">
    <matrixTrace>
      <matrixTimes>
        <matrixReference idx="1"/>
        <matrixVar idx="0"/>
      </matrixTimes>
    </matrixTrace>
  </nl>
  <nl idx="1">
    <matrixTrace>
      <matrixTimes>
        <matrixReference idx="2"/>
        <matrixVar idx="0"/>
      </matrixTimes>
    </matrixTrace>
  </nl>
</nonlinearExpressions>
```

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Sample.osil (p.3)

```
<matrices numberOfMatrices="3">
  <matrix numberOfColumns="7" numberOfRows="7" symmetry="lower" name="F0">
    <blocks numberOfBlocks="2">
      <colOffsets numberOfEl="4">
        <el>0</el>  <el>2</el>  <el>5</el>  <el>7</el>
      </colOffsets>
      <rowOffsets numberOfEl="4">
        <el>0</el>  <el>2</el>  <el>5</el>  <el>7</el>
      </rowOffsets>
      <block blockRowIdx="0" blockColIdx="0">
        <constantElements numberOfValues="3">
          <start>
            <el>0</el>  <el>2</el>  <el>3</el>
          </start>
          <index>
            <el>0</el>  <el mult="2">1</el>
          </index>
          <value>
            <el>2.</el>  <el>1.</el>  <el>2.</el>
          </value>
        </constantElements>
      </block>
```

Sample.osil (p.4)

```
<block blockRowIdx="1" blockColIdx="1">
  <constantElements numberOfValues="4">
    <start>
      <el>0</el>
      <el mult="3" incr="1">2</el>
    </start>
    <index>
      <el>0</el>
      <el>2</el>
      <el>1</el>
      <el>2</el>
    </index>
    <value>
      <el>3.</el>
      <el>1.</el>
      <el>2.</el>
      <el>3.</el>
    </value>
  </constantElements>
</block>
</blocks>
</matrix>
```

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Sample.osil (p.5)

```
<matrix numberOfColumns="7" numberOfRows="7" symmetry="lower" name="F1">
  <blocks numberOfBlocks="2">
    <colOffsets numberOfEl="4">
      <el>0</el>  <el>2</el>  <el>5</el>  <el>7</el>
    </colOffsets>
    <rowOffsets numberOfEl="4">
      <el>0</el>  <el>2</el>  <el>5</el>  <el>7</el>
    </rowOffsets>
    <block blockRowIdx="0" blockColIdx="0">
      <constantElements numberOfValues="3">
        <start>
          <el>0</el>  <el>2</el> <el>3</el>
        </start>
        <index>
          <el>0</el>  <el mult="2">1</el>
        </index>
        <value>
          <el>3.</el>  <el>1.</el>  <el>3.</el>
        </value>
      </constantElements>
    </block>
```

Sample.osil (p.6)

```
      <block blockRowIdx="2" blockColIdx="2">
        <constantElements numberOfValues="1">
          <start>
            <el>0</el>
            <el mult="2">1</el>
          </start>
          <indexes>
            <el>0</el>
          </indexes>
          <values>
            <el>1.</el>
          </values>
        </constantElements>
      </block>
    </blocks>
</matrix>
<matrix numberOfColumns="7" numberOfRows="7" symmetry="lower" name="F2">
    <blocks numberOfBlocks="2">
      <colOffsets numberOfEl="4">
        <el>0</el>  <el>2</el>  <el>5</el>  <el>7</el>
      </colOffsets>
      <rowOffsets numberOfEl="4">
        <el>0</el>  <el>2</el>  <el>5</el>  <el>7</el>
      </rowOffsets>
```

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Sample.osil (p.7)

```
<block blockRowIdx="1" blockColIdx="1">
   <constantElements numberOfValues="4">
      <start>
         <el>0</el>   <el mult="3" incr="1">2</el>
      </start>
      <indexes>
         <el>0</el>   <el>2</el>   <el>1</el>   <el>2</el>
      </indexes>
      <values>
         <el>3.</el>   <el>1.</el>   <el>4.</el>   <el>5.</el>
      </values>
   </constantElements>
</block>
<block blockRowIdx="2" blockColIdx="2">
   <constantElements numberOfValues="1">
      <start>
         <el mult="2">0</el>   <el>1</el>
      </start>
      <indexes>
         <el>1</el>
      </indexes>
      <values>
         <el>1.</el>
      </values>
   </constantElements>
</block>
</blocks>
</matrix>
</matrices>
```

Sample.osil (p.8)

```
    <cones numberOfCones="1">
      <semidefiniteCone  numberOfColumns="7"  numberOfRows="7"/>
    </cones>
    <matrixProgramming>
      <matrixVariables  numberOfMatrixVar="1">
        <matrixVar  numberOfColumns="7"  numberOfRows="7"  lbConeIdx="0"/>
      </matrixVariables>
    </matrixProgramming>
  </instanceData>
</osil>
```

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Comparing SDPA and OSiL (SDPLib problems)

Problem	sdpa	osil	Problem	sdpa	osil	Problem	sdpa	osil	Problem	sdpa	osil
arch0	72,656	271,503	gpp250-2	494,349	189,593	infd2	177,206	171,941	qap7	90,156	243,785
arch2	81,866	287,263	gpp250-3	505,222	199,730	infp1	180,895	173,899	qap8	148,866	359,825
arch4	81,924	286,991	gpp250-4	523,573	214,245	infp2	180,853	174,189	qap9	232,427	514,610
arch8	81,808	286,993	gpp500-1	1,979,166	366,771	maxG11	60,679	559,094	qap10	346,981	709,137
control1	5,776	28,548	gpp500-2	1,989,754	378,570	maxG32	161,010	1,410,904	qpG11	68,352	575,492
control2	45,984	133,359	gpp500-3	2,008,602	395,230	maxG51	153,032	751,737	qpG51	156,139	757,854
control3	160,879	369,212	gpp500-4	2,054,533	432,437	maxG55	557,518	3,641,732	ss30	164,321	378,936
control4	389,895	791,222	hinf1	3,245	16,806	maxG60	705,261	5,036,541	theta1	21,849	67,464
control5	771,116	1,440,187	hinf2	4,256	18,020	mcp100	9,035	71,435	theta2	88,698	313,967
control6	1,342,135	2,395,296	hinf3	4,204	17,969	mcp124-1	7,690	85,255	theta3	206,836	694,336
control7	2,141,737	3,697,616	hinf4	4,147	17,899	mcp124-2	11,197	88,492	theta4	374,526	1,223,092
control8	3,207,058	5,376,565	hinf5	4,192	17,883	mcp124-3	17,379	92,487	theta5	591,344	1,899,325
control9	4,628,035	7,498,138	hinf6	4,216	18,036	mcp124-4	30,505	100,519	theta6	858,179	2,743,012
control10	6,404,548	10,145,798	hinf7	4,218	17,880	mcp250-1	16,852	171,966	thetaG11	267,676	1,692,671
control11	8,616,690	13,357,271	hinf8	4,194	17,989	mcp250-2	23,051	177,234	thetaG51	879,695	4,998,647
equalG11	6,702,617	598,598	hinf9	4,176	17,897	mcp250-3	37,282	187,386	truss1	600	9,906
equalG51	11,056,977	801,060	hinf10	6,577	27,398	mcp250-4	61,361	201,913	truss2	18,232	162,140
gpp100	77,333	76,449	hinf11	14,318	45,001	mcp500-1	33,661	342,084	truss3	2,811	40,633
gpp124-1	117,129	91,468	hinf12	19,830	61,425	mcp500-2	47,104	353,909	truss4	1,205	18,284
gpp124-2	119,867	94,698	hinf13	50,675	107,804	mcp500-3	71,580	370,608	truss5	76,706	759,858
gpp124-3	124,487	98,684	hinf14	84,408	159,704	mcp500-4	131,441	407,846	truss6	53,076	483,319
gpp124-4	134,398	106,709	hinf15	119,449	213,393	qap5	25,986	93,527	truss7	26,131	239,198
gpp250-1	489,484	184,341	infd1	176,811	171,093	qap6	50,796	155,617	truss8	208,801	2,184,696

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Utility routines

- Common operations on matrices
  - Expand constructor list
  - Extract block structure
  - Conversion routines
    - Transpose
    - Change representation
    - Exploit, expand or change symmetry

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Options and results

- Options vs. instance
  - Instance describes what is to be solved
    - Variables, objectives, relationships
    - One instance may be input into many solvers
  - Options explain how to solve it
    - Solver options usually cannot be shared
    - Algorithm tuning
      - e.g., tolerances, pricing and branching rules
    - Job performance
      - e.g., iteration limits, CPU limits
    - System requirements
    - Other, e.g., control of output levels
      - Syntax and meaning may vary
  - Separation of instance and option is not universally agreed upon
    - e.g., starting points, basis information
- Results
    - Solver results
    - Other results
- Matrix-valued options and results

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Instance alterations

- Modifications
  - (e.g., Post-optimality analysis, cut/column generation)
- Real-time data
- Stochastic information (as distinct from deterministic equivalent)
- Disjunctions
- Decomposition

- Common theme: Solver sees a different problem than formulated in the modeling environment
- Often change is incremental
- Smaller problem representation; faster file transmission

Modifications

- Add / delete / replace
  - Variables, objectives, constraints
  - Nonlinear expressions
  - Linear and quadratic coefficients
- Granularity versus ease of handling

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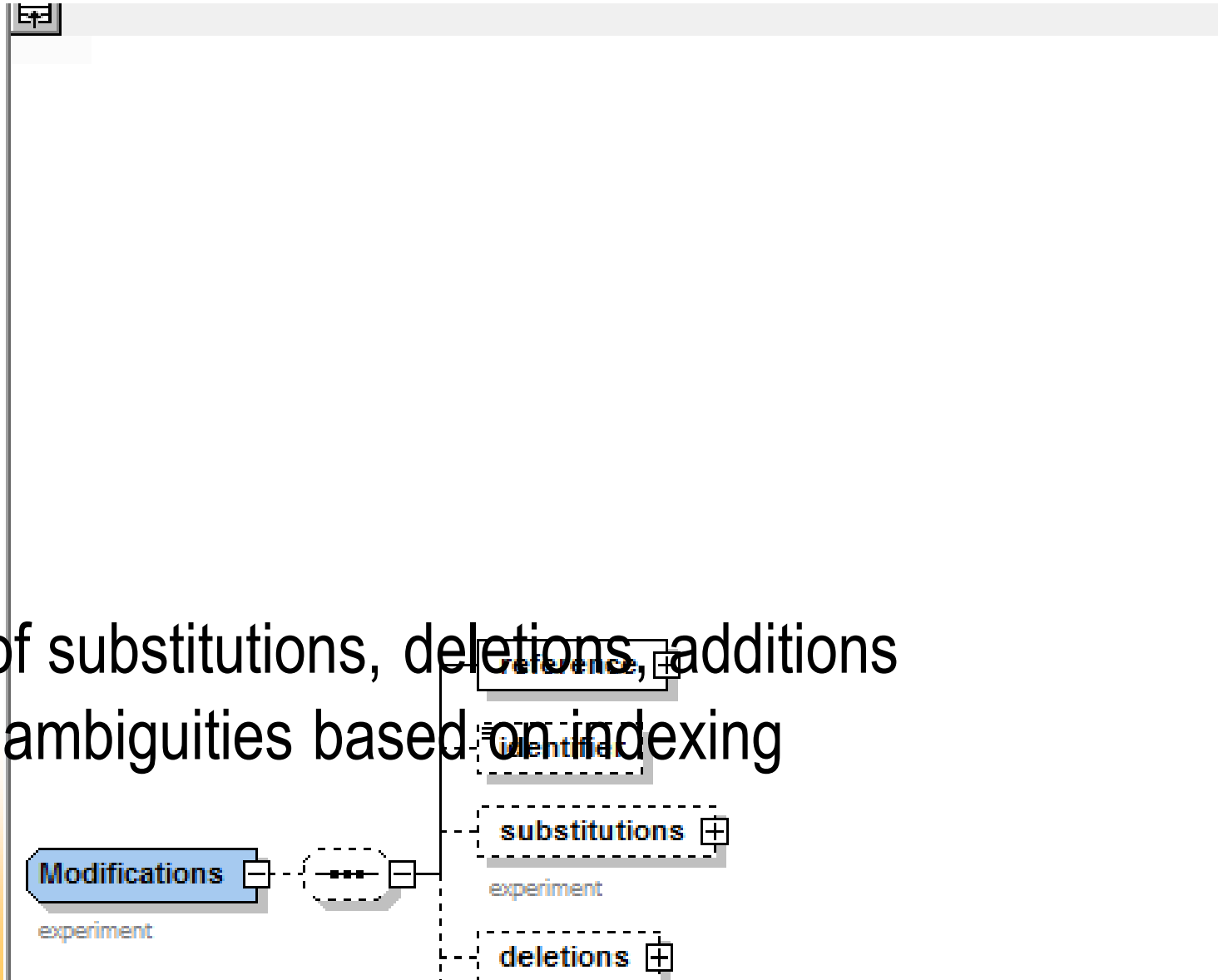
The <modifications> element

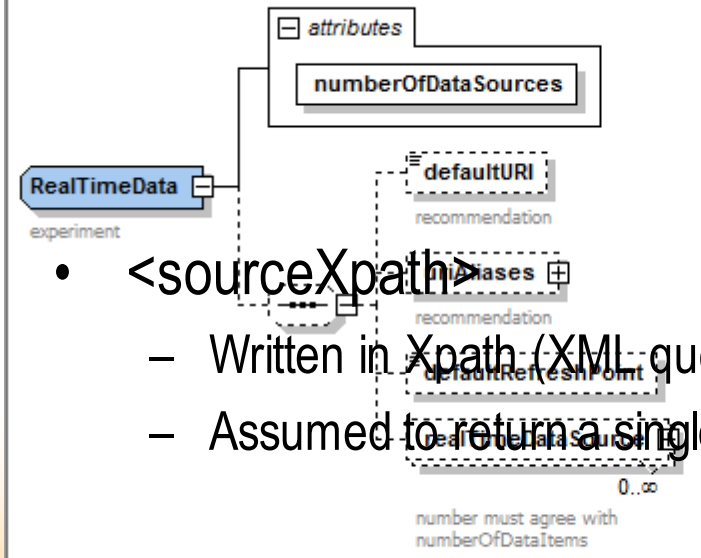
- Order of substitutions, deletions, additions avoids ambiguities based on indexing



Real-time and stochastic data

- Modification only
- Timing of changes
  - Real-time
    - Before instance is transmitted
    - Before remote solver is started
    - Whenever accessed by algorithm
  - Stochastic data
    - Time stage structure
      - Subject to modification?

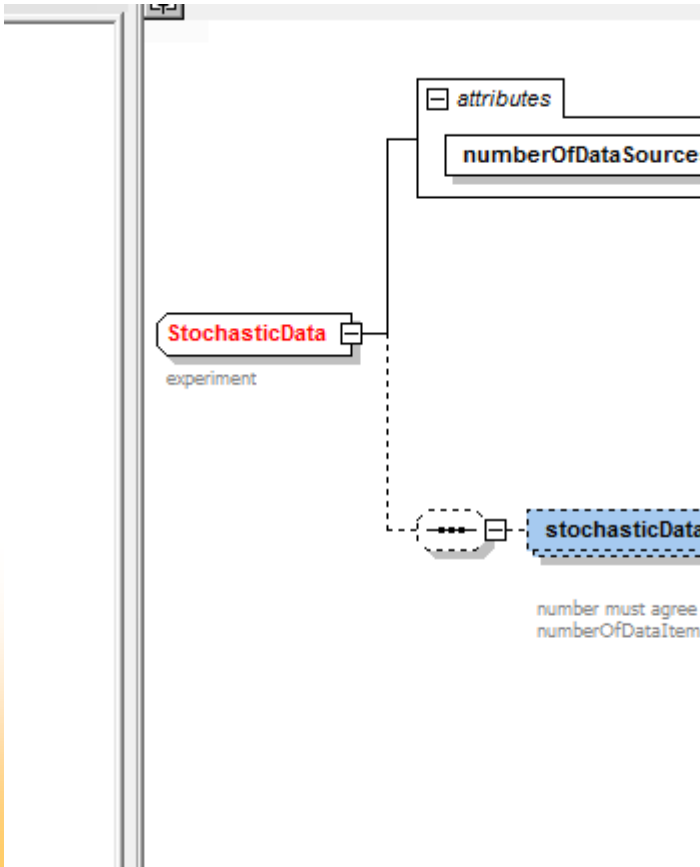
The <realTimeData> element



- <sourceXpath>
  - Written in Xpath (XML query language)
  - Assumed to return a single node from an XML document

Stochastic data

- Scenario-based modelling
  - Condensed representation of the deterministic equivalent
- Distribution-based modelling
  - Distribution information replaces SourceXPath
  - Independent of algorithm



Disjunctions

- variables/objectives/constraints/terms activated or deactivated based on value of other variables
- e.g., fixed charge if $x > 0$
- Often solved by decomposition

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Decomposition

- Extract subproblem from original instance
- Often iterative procedure
- Data and dimensions may be dynamic
- Large portions of solver instance static

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How to get OS

- Download
  - Binaries
    - http://www.coin-or.org/download/binary/OS
      - OS-2.1.1-win32-msvc9.zip
      - OS-2.3.0-linux-x86_64-gcc4.3.2.tgz
  - Stable source
    - http://www.coin-or.org/download/source/OS/
      - OS-2.10.0.tgz
      - OS-2.10.0.zip
  - Development version (using svn)
    - svn co https://projects.coin-or.org/svn/OS/releases/2.10.0
    - svn co https://projects.coin-or.org/svn/OS/trunk
- More information
  - http://www.optimizationservices.org
  - https://projects.coin-or.org/OS
  - Horand.Gassmann@dal.ca

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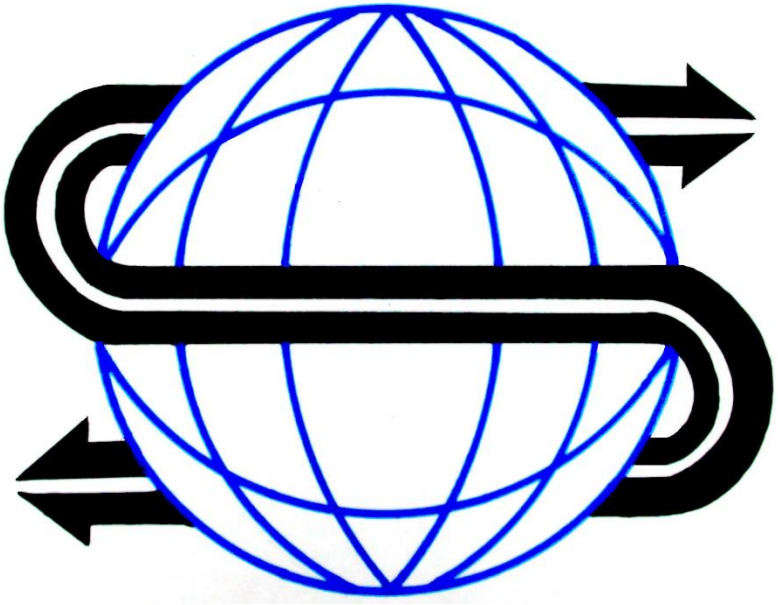
QUESTIONS?



http://www.optimizationservices.org

https://projects.coin-or.org/OS

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